4. Vestigial sideband modulation, in which "almost" the whole of one sideband and a "vestige" of the other sideband are transmitted in a prescribed complementary fashion. VSB modulation requires a channel bandwidth that is intermediate between that required for SSB and DSB-SC systems, and the saving in bandwidth can be significant if modulating signals with large bandwidths are being handled, as in the case of television signals and high-speed digital data.

One final comment is in order. Although the development of the amplitude modulation family has been motivated by its direct relevance to analog communications, many aspects of this branch of modulation theory are equally applicable to digital communications. If, for example, the message signal in Eq. (3.47) for the modulated wave $s(t)$ is restricted to levels of -1 or $+1$ representing a binary "0" and "1" respectively, then we have a basic form of digital modulation known as binary phase-shift-keying (BPSK) that is discussed further in Chapter 7.

ADDITIONAL PROBLEMS

3.17 Throughout the chapter we focused on

$$c(t) = A_c \cos(2\pi f_c t)$$

as the sinusoidal carrier wave. Suppose we choose

$$c(t) = A_c \sin(2\pi f_c t)$$

as the sinusoidal carrier wave. To be consistent, suppose we also define

$$m(t) = A_c \sin(2\pi f_m t)$$

(a) Evaluate the spectrum of the new definition of AM:

$$s(t) = A_c[1 + k_a m(t)] \sin(2\pi f_c t)$$

where k_a is the amplitude sensitivity.

(b) Compare the result derived in part (a) with that studied in Example 3.1.

(c) What difference does the formulation in this problem make to the formulation of modulation theory illustrated in Example 3.1?

3.18. Consider the message signal

$$m(t) = 20 \cos(2\pi t) \text{ volts}$$

and the carrier wave

$$c(t) = 50 \cos(100\pi t) \text{ volts}$$

(a) Sketch (to scale) the resulting AM wave for 75 percent modulation.

(b) Find the power developed across a load of 100 ohms due to this AM wave.

3.19. Using the message signal

$$m(t) = \frac{t}{1 + t^2}$$

determine and sketch the modulated wave for amplitude modulation whose percentage modulation equals the following values:

(a) 50 percent

(b) 100 percent

(c) 125 percent

3.20 Suppose a nonlinear device is available for which the output current i_o and input voltage v_i are related by

$$i_o = a_1 v_i + a_3 v_i^3$$

where a_1 and a_3 are constants. Explain how such a device could be used to provide amplitude modulation. Could such a device also be used for demodulation? Justify your answer.

3.21 Consider the DSB-SC modulated wave obtained by using the sinusoidal modulating wave

$$m(t) = A_m \cos(2\pi f_m t)$$

and the carrier wave

$$c(t) = A_c \cos(2\pi f_c t + \phi)$$

The phase angle ϕ, denoting the phase difference between $c(t)$ and $m(t)$ at time $t = 0$, is variable. Sketch this modulated wave for the following values of ϕ:

(a) $\phi = 0$

(b) $\phi = 45°$

(c) $\phi = 90°$

(d) $\phi = 135°$

Comment on your results.

3.22 Given the nonlinear device described in Problem 3.20, explain how it could be used to provide a product modulator.

3.23 Consider a message signal $m(t)$ with the spectrum shown in Fig. 3.31. The message bandwidth $W = 1$ kHz. This signal is applied to a product modulator, together with a carrier wave $A_c \cos(2\pi f_c t)$, producing the DSB-SC modulated wave $s(t)$. This modulated wave is next applied to a coherent detector. Assuming perfect synchronism between the carrier waves in the modulator and detector, determine the spectrum of the detector output when: (a) the carrier frequency $f_c = 1.25$ kHz and (b) the carrier frequency $f_c = 0.75$ kHz. What is the lowest carrier frequency for which each component of the modulated wave $s(t)$ is uniquely determined by $m(t)$?

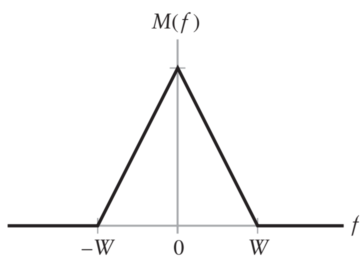

FIGURE 3.31 Problem 3.23

3.24 Consider a composite wave obtained by adding a noncoherent carrier $A_c \cos(2\pi f_c t + \phi)$ to a DSB-SC wave $\cos(2\pi f_c t) m(t)$. This composite wave is applied to an ideal envelope detector. Find the resulting detector output for

(a) $\phi = 0$

(b) $\phi \neq 0$ and $|m(t)| \ll A_c/2$

3.25 A DSB-SC wave is demodulated by applying it to a coherent detector.

(a) Evaluate the effect of a frequency error Δf in the local carrier frequency of the detector, measured with respect to the carrier frequency of the incoming DSB-SC wave.

(b) For the case of a sinusoidal modulating wave, show that because of this frequency error, the demodulated wave exhibits beats at the error frequency. Illustrate your answer with a sketch of this demodulated wave. (A *beat* refers to a signal whose frequency is the difference between the frequencies of two input signals.)

3.26 Consider a pulse of amplitude A and duration T. This pulse is applied to a SSB modulator, producing the modulated wave $s(t)$. Determine the envelope of $s(t)$, and show that this envelope exhibits peaks at the beginning and end of the pulse.

3.27 (**a**) Consider a message signal $m(t)$ containing frequency components at 100, 200, and 400 Hz. This signal is applied to a SSB modulator together with a carrier at 100 kHz, with only the upper sideband retained. In the coherent detector used to recover $m(t)$, the local oscillator supplies a sinusoidal wave of frequency 100.02 kHz. Determine the frequency components of the detector output.

(**b**) Repeat your analysis, assuming that only the lower sideband is transmitted.

3.28 Throughout this chapter, we have expressed the sinusoidal carrier wave in the form

$$c(t) = A_c \cos(2\pi f_c t)$$

where A_c is the carrier amplitude and f_c is the carrier frequency. In Chapter 7 dealing with digital band-pass modulation techniques, we find it more convenient to express the carrier in the form

$$c(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t)$$

where T_b is the duration allotted to the transmission of symbol 1 or symbol 0. Determine the value of carrier amplitude A_c for the energy in $c(t)$ per symbol to equal unity.

ADVANCED PROBLEMS

3.29 For a p-n junction diode, the current i through the diode and the voltage v across it are related by

$$i = I_0\left[\exp\left(-\frac{v}{V_T}\right) - 1\right]$$

where I_0 is the reverse saturation current and V_T is the thermal voltage defined by

$$V_T = \frac{kT}{e}$$

where k is Boltzmann's constant in joules per degree Kelvin, T is the absolute temperature in degrees Kelvin, and e is the charge of an electron. At room temperature, $V_T = 0.026$ volt.

(**a**) Expand i as a power series in v, retaining terms up to v^3.

(**b**) Let

$$v = 0.01\cos(2\pi f_m t) + 0.01\cos(2\pi f_c t) \text{ volts}$$

where $f_m = 1$ kHz and $f_c = 100$ kHz. Determine the spectrum of the resulting diode current i.

(**c**) Specify the bandpass filter required to extract from the diode current an AM wave with carrier frequency f_c.

(**d**) What is the percentage modulation of this AM wave?

3.30 Consider the quadrature-carrier multiplex system of Fig. 3.17. The multiplexed signal $s(t)$ produced at the transmitter output in part (a) of this figure is applied to a communication channel of transfer function $H(f)$. The output of this channel is, in turn, applied to the receiver input in part (b) of Fig. 3.17. Prove that the condition

$$H(f_c + f) = H^*(f_c - f), \qquad \text{for } 0 \le f \le W$$