4.10 Consider an interval Δt of an FM wave $s(t) = A_c \cos[\theta(t)]$ such that $\theta(t)$ satisfies the condition

$$\theta(t + \Delta t) - \theta(t) = \pi$$

Hence, show that if Δt is sufficiently small, the instantaneous frequency of the FM wave inside this interval is approximately given by

$$f_i \approx \frac{1}{2\Delta t}$$

4.11 The sinusoidal modulating wave

$$m(t) = A_m \cos(2\pi f_m t)$$

is applied to a phase modulator with phase sensitivity k_p. The unmodulated carrier wave has frequency f_c and amplitude A_c. Determine the spectrum of the resulting phase-modulated wave, assuming that the maximum phase deviation $\beta = k_p A_m$ does not exceed 0.3 radian.

4.12 A carrier wave is frequency-modulated using a sinusoidal signal of frequency f_m and amplitude A_m.

(a) Determine the values of the modulation index β for which the carrier component of the FM wave is reduced to zero. For this calculation you may use the values of $J_0(\beta)$ given in Appendix 3.

(b) In a certain experiment conducted with $f_m = 1$ kHz and increasing A_m (starting from zero volt), it is found that the carrier component of the FM wave is reduced to zero for the first time when $A_m = 2$ volts. What is the frequency sensitivity of the modulator? What is the value of A_m for which the carrier component is reduced to zero for the second time?

4.13 A carrier wave of frequency 100 MHz is frequency-modulated by a sinusoidal wave of amplitude 20 V and frequency 100 kHz. The frequency sensitivity of the modulator is 25 kHz/V.

(a) Determine the approximate bandwidth of the FM wave, using Carson's rule.

(b) Determine the bandwidth obtained by transmitting only those side-frequencies with amplitudes that exceed one percent of the unmodulated carrier amplitude. Use the universal curve of Fig. 4.9 for this calculation.

(c) Repeat your calculations, assuming that the amplitude of the modulating wave is doubled.

(d) Repeat your calculations, assuming that the modulation frequency is doubled.

4.14 Consider a wide-band PM wave produced by the sinusoidal modulating wave $A_m \cos(2\pi f_m t)$, using a modulator with a phase sensitivity equal to k_p radians per volt.

(a) Show that if the maximum phase deviation of the PM wave is large compared with one radian, the bandwidth of the PM wave varies linearly with the modulation frequency f_m.

(b) Compare this characteristic of a wide-band PM wave with that of a wide-band FM wave.

4.15 Figure 4.19 shows the block diagram of a closed-loop feedback system for the carrier-frequency stabilization of a wide-band frequency modulator. The voltage-controlled oscillator shown in the figure constitutes the frequency modulator. Using the ideas of mixing (i.e., frequency translation) (described in Chapter 3) and frequency discrimination (described in this chapter), discuss how the feedback system of Fig. 4.19 is capable of exploiting the frequency accuracy of the crystal oscillator to stabilize the voltage-controlled oscillator.

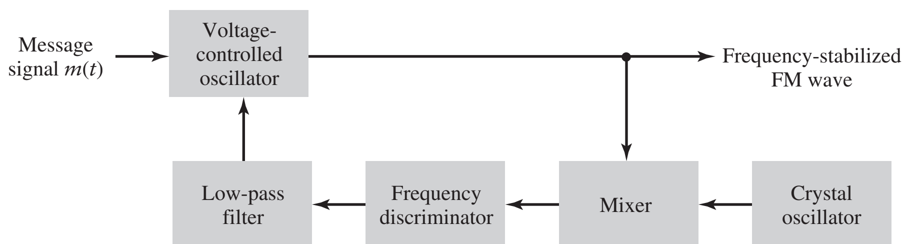

FIGURE 4.19 Problem 4.15

4.16 Consider the frequency demodulation scheme shown in Fig. 4.20 in which the incoming FM wave $s(t)$ is passed through a delay line that produces a phase shift of $-\pi/2$ radians at the carrier frequency f_c. The delay-line output is subtracted from $s(t)$, and the resulting composite wave is then envelope-detected. This demodulator finds application in demodulating FM waves at microwave frequencies. Assuming that

$$s(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_c t)]$$

analyze the operation of this demodulator when the modulation index β is less than unity and the delay T produced by the delay line is sufficiently small to justify making the approximations:

$$\cos(2\pi f_m T) \approx 1$$

and

$$\sin(2\pi f_m T) \approx 2\pi f_m T$$

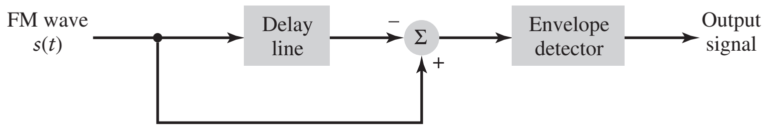

FIGURE 4.20 Problem 4.16

4.17 Consider the following pair of modulating signals:

1. $m_1(t) = \begin{cases} a_1 t + a_0, & t \geq 0 \\ 0, & t = 0 \end{cases}$

2. $m_2(t) = \begin{cases} b_2 t^2 + b_1 t + b_0, & t \geq 0 \\ 0, & t = 0 \end{cases}$

where the *a*s and the *b*s are constant parameters.

Signal 1 is applied to a frequency modulator, while signal 2 is applied to a phase modulator. Determine the conditions for which the outputs of these two angle modulators are exactly the same.

4.18 In this problem, we work on the specifications of a superheterodyne FM receiver listed in Table 3.2. In particular, given those specifications, do the following work:

(a) Determine the range of frequencies provided by the local oscillator of the receiver in order to accommodate the RF carrier range 88-108 MHz.

(b) Determine the corresponding range of image frequencies.

ADVANCED PROBLEMS

4.19 The instantaneous frequency of a sinusoidal wave is equal to $f_c + \Delta f$ for $|t| < T/2$ and f_c for $|t| > T/2$. Determine the spectrum of this frequency-modulated wave. *Hint*: Divide up the time interval of interest into three nonoverlapping regions:

(i) $-\infty < t < -T/2$

(ii) $-T/2 \leq t \leq T/2$

(iii) $T/2 < t < \infty$

4.20 Figure 4.21 shows the block diagram of a *real-time spectrum analyzer* working on the principle of frequency modulation. The given signal $g(t)$ and a frequency-modulated signal $s(t)$ are applied to a multiplier and the output $g(t)s(t)$ is fed into a filter of impulse $h(t)$. The $s(t)$ and $h(t)$ are linear FM signals whose instantaneous frequencies vary at opposite rates, as shown by

$$s(t) = \cos(2\pi f_c t + \pi k t^2)$$