(iv) We showed how quadrature modulation schemes such as QPSK and QAM provide the same performance as their one-dimensional counterparts of BPSK and PAM, due to the orthogonality of the in-phase and quadrature components of the modulated signals. In particular, we showed how QPSK modulation provides the same BER as BPSK for the same E_b/N_0 but provides double the throughput.

(v) We also showed that antipodal strategies such as BPSK are more power efficient than orthogonal transmission strategies such as on–off signaling and FSK.

(vi) We introduced the concept of non-coherent detection when we illustrated that BPSK could be detected using a new approach where the transmitted bits are differentially encoded. The simplicity of this detection technique results in a small BER performance penalty.

The signal-space concepts introduced in Chapter 7 provide an intuitive geometrical interpretation of the relative performance of the different coherent digital modulation strategies in noise. Finally, we closed the chapter with a brief introduction of forward error correction coding, which can be combined with any of the above digital modulation strategies to reduce the power required to achieve the same bit error rate performance—however, there is usually a power and bandwidth tradeoff that must be made when using forward-error-correction coding in the design of digital communication systems.

ADDITIONAL PROBLEMS

10.11 A communications system that transmits single isolated pulses is subject to multipath such that, if the transmitted pulse is $p(t)$ of lenght T, the received signal is

$$s(t) = p(t) + \alpha p(t - \tau)$$

Assuming that α and τ are known, determine the optimum receiver filter for signal in the presence of white Gaussian noise of power spectral density $N_0/2$. What is the post-detection SNR at the output of this filter?

10.12 The impulse response corresponding to a root-raised cosine spectrum, normalized to satisfy Eq. (10.10), is given by

$$g(t) = \left(\frac{4\alpha}{\pi}\right) \frac{\cos\left[\frac{(1+\alpha)\pi t}{T}\right] + \frac{T}{4\alpha t}\sin\left[\frac{(1-\alpha)\pi t}{T}\right]}{1 - \left(\frac{4\alpha t}{T}\right)^2}$$

where $T = 1/(2B_0)$ is the symbol period and α is the roll-off factor. Obtain a discrete-time representation of this impulse response by sampling it at $t = 0.1nT$ for integer n such that $-3T < t < 3T$. Numerically approximate matched filtering (e.g. with MatLab) by performing the discrete-time convolution

$$q_k = 0.1 \sum_{n=-60}^{60} g_n g_{k-n}$$

where $g_n = g(0.1nT)$. What is the value of $q_k = q(0.1kT)$ for $k = \pm 20, \pm 10$, and 0?

10.13 Determine the discrete-time autocorrelation function of the noise sequence $\{N_k\}$ defined by Eq. (10.34)

$$N_k = \int_{-\infty}^{\infty} p(kT - t)w(t)dt$$

where $w(t)$ is a white Gaussian noise process and the pulse $p(t)$ corresponds to a root-raised cosine spectrum. How are the noise samples corresponding to adjacent bit intervals related?

10.14 Draw the Gray-encoded constellation (signal-space diagram) for 16-QAM and for 64-QAM. Can you suggest a constellation for 32-QAM?

10.15 Write the defining equation for a QAM-modulated signal. Based on the discussion of QPSK and multi-level PAM, draw the block diagram for a coherent QAM receiver.

10.16 Show that the if T is a multiple of the period of f_c, then the terms $\sin(2\pi f_c t)$ and $\cos(2\pi f_c t)$ are orthogonal over the interval $[t_0, T + t_0]$.

10.17 For a rectangular pulse shape, by how much does the null-to-null transmission bandwidth increase, if the transmission rate is increased by a factor of three?

10.18 Under the bandpass assumptions, determine the conditions under which the two signals $\cos(2\pi f_0 t)$ and $\cos(2\pi f_1 t)$ are orthogonal over the interval from 0 to T.

10.19 Encode the sequence 1101 with a Hamming (7, 4) block code.

10.20 The Hamming (7, 4) encoded sequence 1001000 was received. If the number of transmission errors is less than two, what was the transmitted sequence?

10.21 A Hamming (15, 11) block code is applied to a BPSK transmission scheme. Compare the block error rate performance of the uncoded and coded systems. Explain how this would differ if the modulation strategy was QPSK.

Advanced Problems

10.22 Show that the choice $\gamma = \mu/2$ minimizes the probability of error given by Eq. (10.26). *Hint:* The Q-function is continuously differentiable.

10.23 For M-ary PAM,

(**a**) Show that the formula for probability of error, namely,

$$P_e = 2\left(\frac{M-1}{M}\right)Q\left(\frac{A}{\sigma}\right)$$

holds for $M = 2$, 3, and 4. By mathematical induction, show that it holds for all M.

(**b**) Show the formula for average power, namely,

$$P = \frac{(M^2 - 1)A^2}{3}$$

holds for $M = 2$, and 3. Show it holds for all M.

10.24 Consider binary FSK transmission where $(f_1 - f_2)T$ is not an integer.

(**a**) What is the mean output of the upper correlator of Fig. 10.12, if a 1 is transmitted? What is the mean output of the lower correlator?

(**b**) Are the random variables N_1 and N_2 independent under these conditions? What is the variance of $N_1 - N_2$?

(**c**) Describe the properties of the random variable D of Fig. 10.12 in this case.

10.25 Show that the noise variance of the in-phase component $n_I(t)$ of the band-pass noise is the same as the band-pass noise $n(t)$ variance; that is, for a band-pass noise of bandwidth B_N, we have

$$\mathbf{E}[n_I^2(t)] = N_0 B_N$$

10.26 In this problem, we investigate the effects when transmit and receive filters do not combine to form an ISI-free pulse shape. To be specific, data are transmitted at baseband using binary PAM with an exponential pulse shape $g(t) = \exp(t/T)u(t)$ where T is the symbol period (see Example 2.2). The receiver detects the data using an integrate-and-dump detector.

(**a**) With data symbols represented as ± 1, what is magnitude of the signal component at the output of the detector?

(**b**) What is the worst case magnitude of the intersymbol interference at the output of the detector. (Assume the data stream has infinite length.) Using the value obtained in part (a) as a reference, by what percentage is the eye opening reduced by this interference.

(c) What is the rms magnitude of the intersymbol interference at the output of the detector? If this interference is treated as equivalent to noise, what is the equivalent signal-to-noise ratio at the output of the detector? Comment on how this would affect bit error rate performance of this system when there is also receiver noise present.

10.27 A BPSK signal is applied to a matched-filter receiver that lacks perfect phase synchronization with the transmitter. Specifically, it is supplied with a local carrier whose phase differs from that of the carrier used in the transmitter by ϕ radians.

(a) Determine the effect of the phase error ϕ on the average probability of error of this receiver.

(b) As a check on the formula derived in part (a), show that when the phase error is zero the formula reduces to the same form as in Eq. (10.44).

10.28 A binary FSK system transmits data at the rate of 2.5 times megabits per second. During the course of transmission, white Gaussian noise of zero mean and power spectral density 10^{-20} watts per hertz is added to the signal. In the absence of noise, the amplitude of the received signal is 1 μV across 50 ohm impedance. Determine the average probability of error assuming coherent detection of the binary FSK signal.

10.29 One of the simplest forms of forward error correction code is the repetition code. With an N-repetition code, the same bit is sent N times, and the decoder decides in favor of the bit that is detected on the majority of trials (assuming N is odd). For a BPSK transmission scheme, determine the BER performance of a 3-repetition code.

■ COMPUTER EXPERIMENTS

10.30 In this experiment, we simulate the performance of bipolar signaling in additive white Gaussian noise. The MATLAB script included in Appendix 7 for this experiment does the following:

- It generates a random sequence with rectangular pulse shaping.
- It adds Gaussian noise.
- It detects the data with a simulated integrate-and-dump detector.

With this MATLAB script:

(a) Compute the spectrum of the transmitted signal and compare to the theoretical.

(b) Explain the computation of the noise variance given an E_b/N_0 ratio.

(c) Confirm the theoretically predicted bit error rate for E_b/N_0 from 0 to 10 dB.

10.31 In this experiment, we simulate the performance of bipolar signaling in additive white Gaussian noise but with root-raised-cosine pulse shaping. A MATLAB script is included in Appendix 7 for doing this simulation. Hence, do the following:

(a) Compute the spectrum of the transmitted signal and compare to the theoretical. Also compare to the transmit spectrum with rectangular pulse shaping.

(b) Plot the eye diagram of the received signal under no noise conditions. Explain the relationship of the eye opening to the bit error rate performance.

(c) Confirm the theoretically predicted bit error rate for E_b/N_0 from 0 to 10 dB.

10.32 In this experiment, we simulate the effect of various mismatches in the communication system and their effects on performance. In particular, modify the MATLAB scripts of the two preceding problems to do the following:

(a) Simulate the performance of a system using rectangular pulse-shaping at the transmitter and raised cosine pulse shaping at the receiver. Comment on the performance degradation.

(b) In the case of matched root-raised cosine filtering, include a complex phase rotation $\exp(j\theta)$ in the channel. Plot the resulting eye diagram for θ being the equivalent of 5°, 10°, 20°, and 45°. compare to the case of $\theta = 0°$. Do likewise for the BER performance. What modification to the theoretical BER formula would accurately model this behavior?