Department of Electrical Engineering, NKNU

$\boxed{\text{Quiz 3}}$ *Signals and Systems* April 9, 2024

ID:										Name:	Time: 10 minutes	Max. Marks – 40

1. Determine whether the signal given by the following expression is periodic.

$$x(t) = e^{j5\pi/7} + 2\cos\left(\frac{3t}{14} + \frac{\pi}{6}\right) - \sin\left(\frac{t}{3}\right) + \cos\left(\frac{7t}{5} - \sqrt{2}\,\pi\right)$$

Marks - 15

If yes, find the fundamental period.

$T_{01} = \frac{2\pi}{5/7} = \frac{14\pi}{5}$

$T_{02} = \frac{2\pi}{3/14} = \frac{28\pi}{3}$

$T_{03} = \frac{2\pi}{1/3} = 6\pi$

$T_{04} = \frac{2\pi}{7/5} = \frac{10\pi}{7}$

$\frac{T_{01}}{T_{02}} = \frac{\frac{14\pi}{5}}{\frac{28\pi}{3}} = \frac{3}{10}$

$\frac{T_{01}}{T_{03}} = \frac{\frac{14\pi}{5}}{6\pi} = \frac{7}{15}$

$\frac{T_{01}}{T_{04}} = \frac{\frac{14\pi}{5}}{\frac{10\pi}{7}} = \frac{49}{25}$

$5 \,\underline{|\; 10 \quad 15 \quad 25}$
$\quad\; 2 \quad 3 \quad 5$

$K = lcm(10, 15, 25)$
$= 150$

$T_0 = k \times T_{01} = 150 \times \frac{14\pi}{5} = \underline{420\pi}$ ※

2. (a) Write the system model for the system depicted in Fig. 1.

Marks - 25

$y(t) = x(t) + \frac{1}{5}x(t-0.1)$

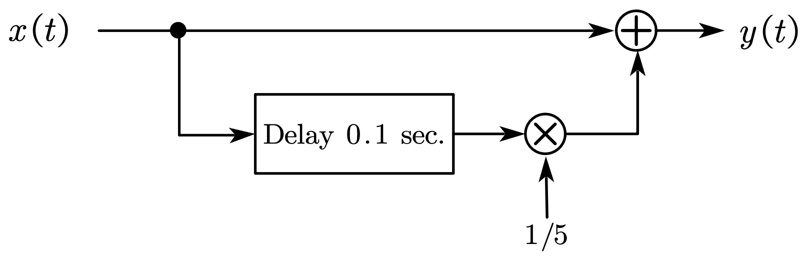

Fig. 1.

(b) Is the system **memoryless**? Show the reason mathematically.

$t_0 \quad t_0 \quad t_0 - 0.1$

$t_0 \neq t_0 - 0.1$ NOT memoryless

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(c) Is the system **causal**? Show the reason mathematically.

$$y(t) = x(t) + \frac{1}{5}x(t-0.1)$$

$$t_0 \geq t_0 \quad t_0-0.1$$

<u>causal</u>

(d) Is the system **BIBO stable**? Show the reason mathematically.

$$|x(t)| \leq M$$

$$|y(t)| = |x(t) + \frac{1}{5}x(t-0.1)|$$

$$\leq |x(t)| + \frac{1}{5}|x(t-0.1)|$$

$$\leq \frac{6}{5}M = R, \ \forall t.$$

<u>BIBO stable</u>

(e) Is the system **linear**? Show the reason mathematically.

$$x_1(t) \longrightarrow y_1(t)$$

$$x_2(t) \longrightarrow y_2(t)$$

$$a_1 x_1(t) + a_2 x_2(t) \longrightarrow a_1 x_1(t) + \frac{a_1}{5}x_1(t-0.1)$$

$$+ a_2 x_2(t) + \frac{a_2}{5}x_2(t-0.1)$$

$$= a_1 y_1(t) + a_2 y_2(t)$$

<u>Linear</u>

(f) Is the system **time-invariant**? Show the reason mathematically.

Input delay $y_d(t) = y(t)\Big|_{x(t-t_0)} = x(t-t_0) + \frac{1}{5}x(t-t_0-0.1)$

Output delay $y(t-t_0) = x(t-t_0) + \frac{1}{5}x(t-t_0-0.1)$

$$y_d(t) = y(t-t_0)$$

<u>Time-Invariant</u>

(g) Is it an **LTI system**?

Yes. It is both linear and time-invariant.

(*) Not invertible.

For an invertiable system, if $y_1(t) = y_2(t)$, then we should show $x_1(t) = x_2(t)$, $\forall t$.

$x_1(t) + 0.2x_1(t-0.1) = x_2(t) + 0.2x_2(t-0.1)$

$x_1(t) - x_2(t) = 0.2[x_1(t-0.1) - x_2(t-0.1)]$

The above equation cannot guarantee that $x_1(t) - x_2(t) = 0$, $\forall t$.