PROBLEM 3.1

(a) $y(t) = h(t) * x(t) = \int_{-\infty} x(\tau)h(t-\tau)d\tau$

(i) $x(t) = u(t-2)$

$$y(t) = \int_{-\infty}^{\infty} u(\tau-2)u(t-\tau)d\tau = \int_{2}^{t} 1 d\tau$$

$$y(t) = \begin{cases} 0, & t < 2 \\ t-2, & t \ge 2 \end{cases} = (t-2)u(t-2)$$

(ii) $x(t) = e^{-2t}u(t)$

$$y(t) = \int_{-\infty}^{\infty} e^{-2\tau}u(\tau)u(t-\tau) = \int_{0} e^{-2\tau}d\tau$$

$$y(t) = \tfrac{1}{2}(1-e^{-2t})u(t)$$

(iii) $x(t) = tu(t)$

$$y(t) = \int_{-\infty}^{\infty} \tau u(\tau)u(t-\tau)d\tau = \int_{0}^{t} \tau d\tau = \tfrac{1}{2}\tau^2\Big|_{0}^{t}$$

$$y(t) = \tfrac{1}{2}t^2u(t)$$

(iv) $x(t) = (t+1)u(t+1)$

$$y(t) = \int_{-\infty}^{\infty} (\tau+1)u(\tau+1)u(t-\tau)d\tau = \int_{-1}^{t} (\tau+1)d\tau$$

$$= \left(\tfrac{1}{2}\tau^2 + \tau\right)\Big|_{-1}^{t} = \tfrac{1}{2}t^2 + t - \tfrac{1}{2} + 1$$

$$y(t) = \left(\tfrac{1}{2}t^2 + t + \tfrac{1}{2}\right)u(t+1)$$

(v) $x(t) = e^{-2|t|}$; $y(t) = \int_{-\infty}^{\infty} e^{-2|\tau|}u(t-\tau\ \ \tau$

$$y(t) = \int_{-\infty}^{t} e^{-2|\tau|}d\tau$$

for $t<0$, $y(t) = \int_{-\infty}^{0} e^{2\tau}d\tau = \tfrac{1}{2}e^{2t},\ t<0$

for $t \ge 0$, $y(t) = \int_{-\infty}^{0} e^{2\tau}d\tau + \int_{0}^{t} e^{-2\tau}d\tau = 1 - \tfrac{1}{2}e^{-2t},\ t \ge 0$

Problem 3.1 (Continued)

(a)

(vi) $x(t) = t^2 u(t) \Rightarrow y(t) = \int_0^t \tau^2 d\tau = \frac{1}{3}t^3 u(t)$

(vii) $x(t) = (t-1)u(t-1)$

$$y(t) = \int_1^t (\tau - 1)\, d\tau = \left(\frac{1}{2}\tau^2 - \tau\right)\Big|_1^t = \frac{1}{2}t^2 - t - \frac{1}{2} + 1$$

$$y(t) = \left(\frac{1}{2}t^2 - t + \frac{1}{2}\right)u(t-1)$$

(viii) $x(t) = u(t) - u(t-5)$

$$y(t) = \int_0^t d\tau - \int_5^t d\tau = \tau\Big|_0^t - \tau\Big|_5^t$$

$$y(t) = t - t + 5 = 5, \quad t \geq 5$$

$$y(t) = t, \quad 0 \leq t < 5$$

$$y(t) = 0, \quad t < 0$$

$$\therefore y(t) = t\,u(t) + (5-t)u(t-5)$$

(b)

(i) $y(t) = \int_{-\infty}^t x(\tau) = \int_{-\infty}^t u(\tau - 2)\, d\tau = \int_2^t d\tau$

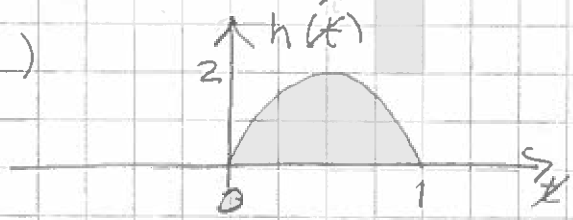
$$y(t) = \begin{cases} 0, & t < 2 \\ t-2, & t \geq 2 \end{cases} = (t-2)u(t-2)$$

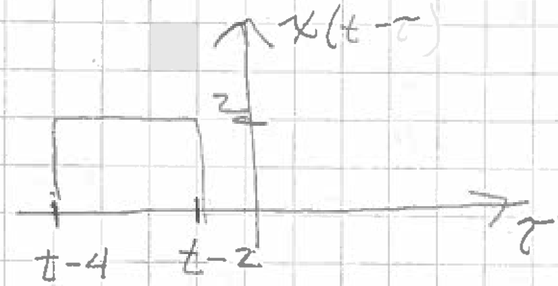
(ii) $y(t) = \int_{-\infty}^t e^{-2\tau} u(\tau)\, d\tau = \int_0^t e^{-2\tau} d\tau = \frac{-1}{2} e^{-2\tau}\Big|_0^t, \; t \geq 0$

$$y(t) = -\frac{1}{2}e^{-2t} + 1/2, \quad t \geq 0 = \frac{1}{2}\left(1 - e^{-2t}\right)u(t)$$

PROBLEM 3.2

(a)





$$y(t) = 4\int_0^{t-2} \sin \pi\tau \, d\tau = -\frac{4}{\pi} \cos \pi\tau \Big|_0^{t-2}$$

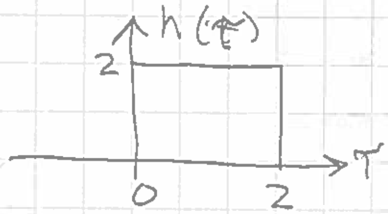
$$= \frac{4}{\pi}\left[1 - \cos \pi(t-2)\right], \quad 2 < t < 3$$

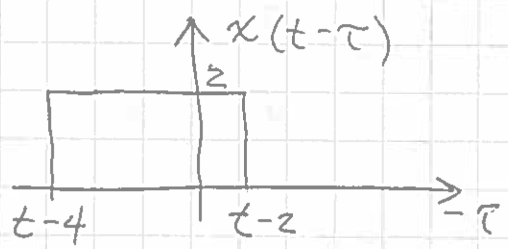
$$y(t) = 4\int_0^1 \sin \pi t \, dt = -\frac{4}{\pi} \cos \pi t \Big|_0^1 = \frac{8}{\pi}, \quad 3 \le t \le 4$$

$$y(t) = 4\int_{t-4}^1 \sin \pi\tau \, d\tau = \frac{4}{\pi}\left[\cos \pi(t-4) + 1\right], \quad 4 < t < 5$$

$$y(t) = 0, \quad t < 2 \text{ and } t > 5$$

(b)





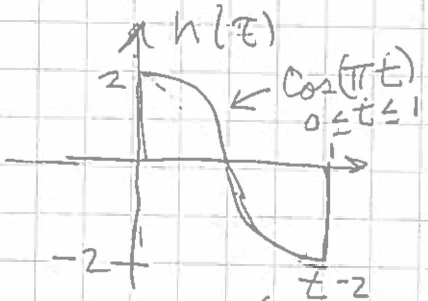
$$y(t) = 4\int_0^{t-2} d\tau = 4\tau \Big|_0^{t-2} = 4t - 8, \quad 2 \le t < 4$$

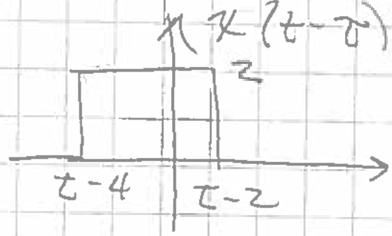
$$y(t) = 4\int_{t-4}^{2} d\tau = 4\tau \Big|_{t-4}^{2} = 24 - 4t, \quad 4 \le t < 6$$

$$y(t) = 0, \quad t < 2, \text{ and } t > 6$$

Problem 3.2 (continued)

(c)





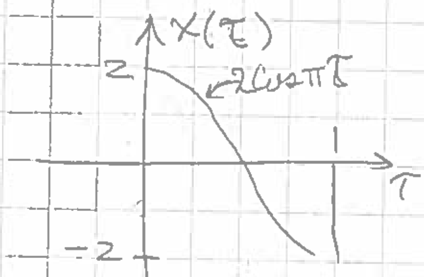
$$y(t) = 4\int_0^{t-2} \cos(\pi\tau)d\tau = \frac{4}{\pi}\sin\pi\tau\Big|_0^{t-2} = \frac{4}{\pi}\sin\pi(t-2),\ 0<t<1$$

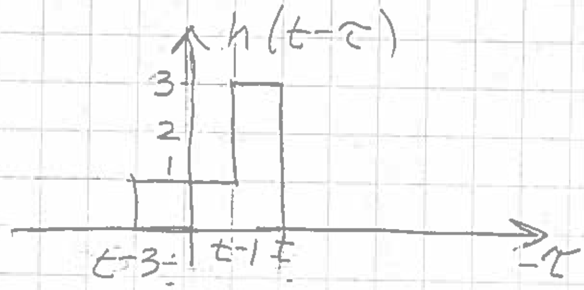
$$y(t) = 4\int_0^1 \cos\pi\tau\, d\tau = \frac{4}{\pi}\sin\pi t\Big|_0^1 = 0,\ 1\le t<4$$

$$y(t) = 4\int_{t-4}^1 \cos\pi\tau\, d\tau = \frac{4}{\pi}\sin\pi\tau\Big|_{t-4}^1 = \frac{-4}{\pi}\sin\pi(t-4),\ 4<t<5$$

$y(t) = 0,\ t<0$ and $t>5$

(d)



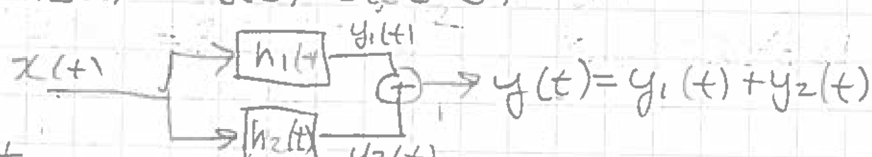


let $h(t) = h_1(t) + h_2(t)$

$h_1(t) = 2[u(t) - u(t-1)]$

$h_2(t) = u(t) - u(t-3)$

CONSIDER:



$$y_1(t) = \int_0^t 4\cos\pi\tau\, d\tau = \frac{4}{\pi}\sin\pi\tau\Big|_0^t = \frac{4}{\pi}\sin\pi t,\ 0\le t<1$$

$$= \int_{t-1}^1 4\cos\pi\tau\, d\tau = \frac{4}{\pi}\sin\pi\tau\Big|_{t-1}^1 = -\frac{4}{\pi}\sin\pi(t-1),\ 1<t<2$$

$y_1(t) = 0,\ t<0,\ t>2$

PROBLEM 3.2 (d) (continued)

$$y_2(t) = \int_0^t 2\cos\pi\tau\, d\tau = \frac{2}{\pi}\sin\pi\tau \Big|_0^t = \frac{2}{\pi}\sin\pi t, \quad 0<t<1$$

$$= \int_0^1 2\cos\pi\tau\, d\tau = \frac{2}{\pi}\sin\pi\tau \Big|_0^1 = 0, \quad 1<t<2$$

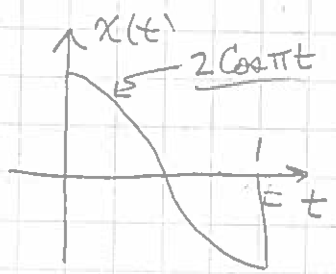
$$= \int_{t-2}^1 2\cos\pi\tau\, d\tau = \frac{2}{\pi}\sin\pi\tau \Big|_{t-2}^1 = -\frac{2}{\pi}\sin\pi(t-2), \quad 2<t<3$$

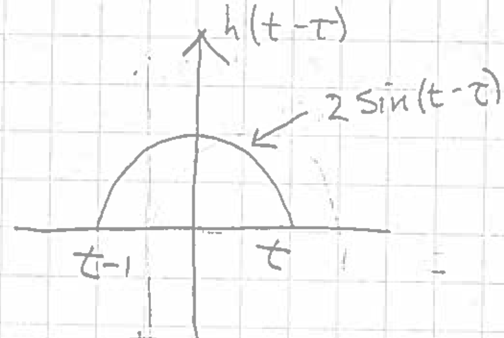
$$y_2(t) = 0, \quad t<0,\ t>3$$

$$y(t) = y_1(t) + y_2(t)$$

$$y(t) = \begin{cases} 0, & t<0 \\ \frac{6}{\pi}\sin\pi t, & 0<t<1 \\ -4\sin\pi(t-1), & 1<t<2 \\ -2\sin\pi(t-2), & 2<t<3 \\ 0, & t>3 \end{cases}$$

(e)





$\underline{0<t<1}$

$$y(t) = \int_0^t 4\sin\pi\tau\cos\pi\tau\, d\tau = 2\int_0^t \sin 2\pi\tau\, d\tau$$

$$= \frac{-2}{2\pi}\cos 2\pi\tau \Big|_0^t = \underline{1 - \frac{1}{\pi}\cos 2\pi t}, \quad 0<t<1$$

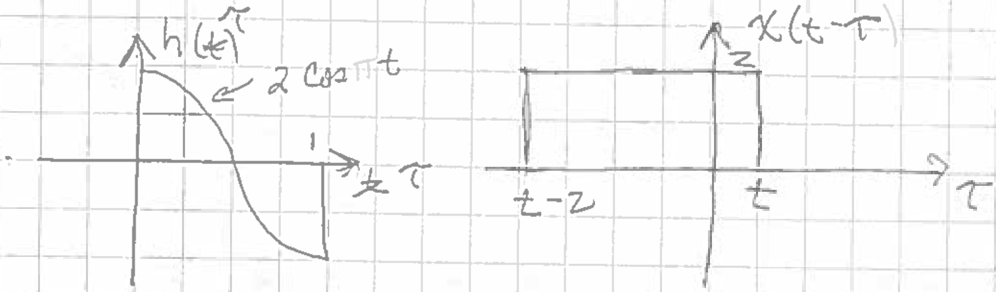
$\underline{1<t<2}$

$$y(t) = \int_{t-1}^1 2\sin 2\pi t\, d\tau = -\frac{1}{\pi}\cos\pi\tau \Big|_{t-1}^1 = \underline{-1 + \frac{1}{\pi}\cos\pi(t-1)}, \quad 1<t<2$$

$$y(t) = 0, \quad t<0,\ t>2$$

PROBLEM 3.2 (continued)

(f)



$y(t) = 0, \quad t < 0$

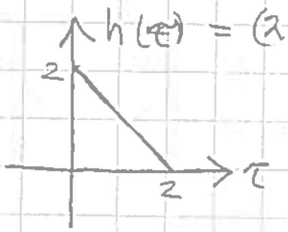
$\underline{0 < t < 1}$: $\quad y(t) = \displaystyle\int_0^t 4\cos \pi\tau \, d\tau = \frac{4}{\pi}\sin \pi\tau \Big|_0^t = \frac{4}{\pi}\sin \pi t, \quad \underline{0<t<1}$

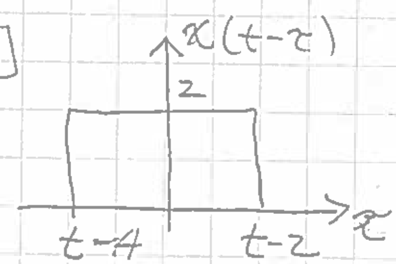
$\underline{1 < t < 2}$: $\quad y(t) = \displaystyle\int_0^1 4\cos \pi\tau \, d\tau = \frac{4}{\pi}\sin \pi\tau \Big|_0^1 = 0$

$\underline{2 < t < 3}$: $\quad y(t) = \displaystyle\int_{t-2}^1 4\cos \pi\tau \, d\tau = \frac{4}{\pi}\sin \pi\tau \Big|_{t-2}^1 = -\frac{4}{\pi}\sin \pi(t-2), \quad 2<t<3$

$y(t) = 0, \quad t > 3$

(g) $h(\tau) = (2-\tau)[u(\tau) - u(\tau-2)]$





$y(t) = 0, \quad t < 2$

$\underline{2 < t < 4}$: $\quad y(t) = 2\displaystyle\int_0^{t-2} (2-\tau)\, d\tau = (4\tau - \tau^2)\Big|_0^{t-2}$

$$y(t) = 4(t-2) - (t-2)^2 = -t^2 + 8t - 12, \quad 2<t<4$$

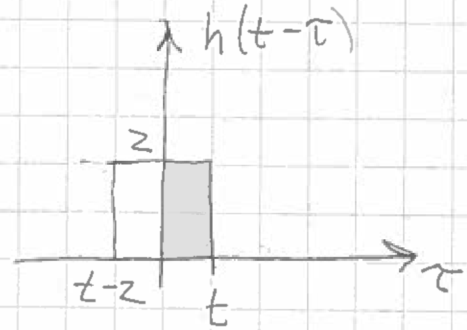
$\underline{4 < t < 6}$: $\quad y(t) = 2\displaystyle\int_{t-4}^2 (2-\tau)\, d\tau = (4\tau - \tau^2)\Big|_{t-4}^2 = (8-4) - [4t - 16 - (t-4)^2]$

$$= 4 - 4t + 16 + t^2 - 8t + 16 = t^2 - 12t + 36, \quad 4<t<6$$

$y(t) = 0, \quad t > 6$

PROBLEM 3.2 (continued)

(h)



$y(t) = 0, \; t < 0$

$\underline{0 < t < 2}$: $y(t) = 2\int_0^t (2-\tau)\,d\tau = 4\tau - \tau^2 \Big|_0^t = 4t - t^2$

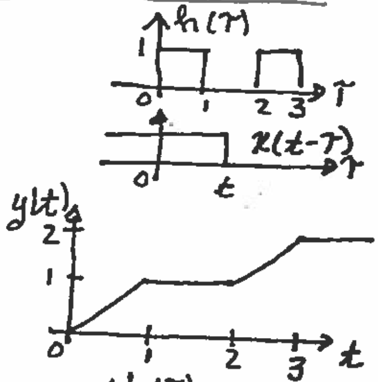
$\underline{2 < t < 4}$: $y(t) = 2\int_{t-2}^{2} (2-\tau)\,d\tau = 4\tau - \tau^2 \Big|_{t-2}^{2}$

$y(t) = (8-4) - \left[4(t-2) - (t-2)^2\right] = t^2 - 8t + 16, \; \underline{2 < t < 4}$

$y(t) = 0, \; t > 4$

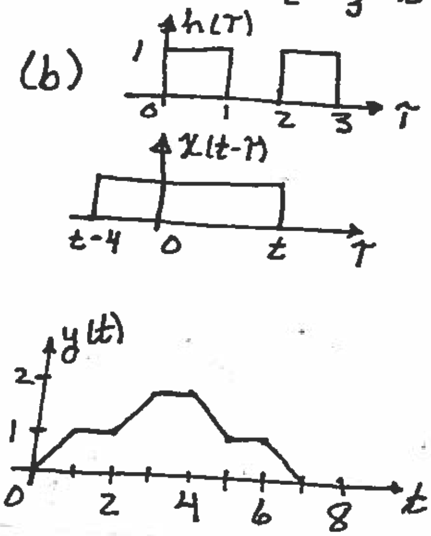
PROBLEM 3.3

(a)



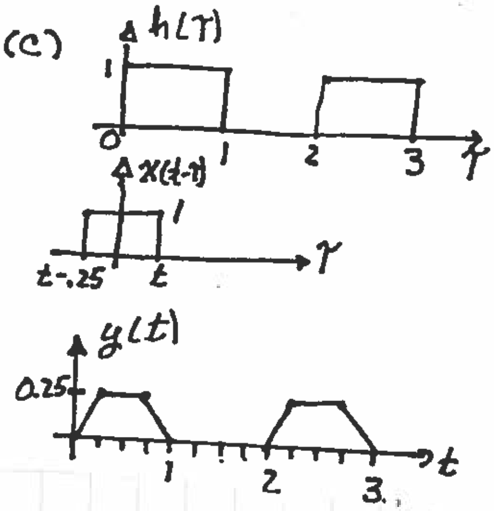
$$
\begin{aligned}
y(t) &= 0, \quad t<0 \\
&= t, \quad 0<t<1 \\
&= 1, \quad 1<t<2 \\
&= 1+(t-2)=t-1, \quad 2<t<3 \\
&= 2, \quad 3<t
\end{aligned}
$$

(b)



range	y(t)
$t<0$	0
$0<t<1$	t
$1<t<2$	1
$2<t<3$	$1-t$
$3<t<4$	2
$4<t<5$	$2-(t-4)=6-t$
$5<t<6$	1
$6<t<7$	$1-(t-6)=7-t$
$0<t,$	0

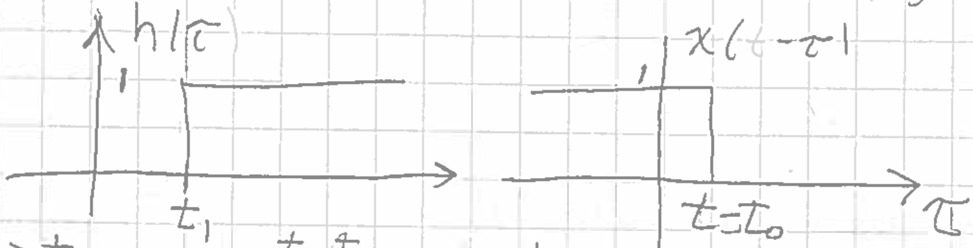
(c)



range	y(t)
$t<0$	0
$0<t<.25$	t
$.25<t<1$	0.25
$1<t<1.25$	$0.25-(t-1)=1.25-t$
$1.25<t<2$	0
$2<t<2.25$	$t-2$
$2.25<t<3$	0.25
$3<t<3.25$	$3.25-t$
$3.25<t$	0

PROBLEM 3.4

$x(t) = u(t-t_0)$, $h(t) = u(t-t_1)$, $t_1 > t_0$

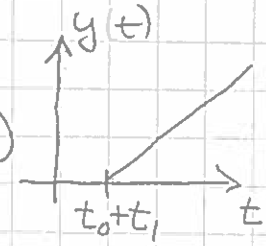


$\underline{t-t_0 \geq t_1}$

$$y(t) = \int_{t_1}^{t-t_0} d\tau = \tau \Big|_{t_1}^{t-t_0} = t-t_0-t_1 \ , \quad t > t_0+t_1$$

$y(t) = 0 \ , \quad t < t_0+t_1$

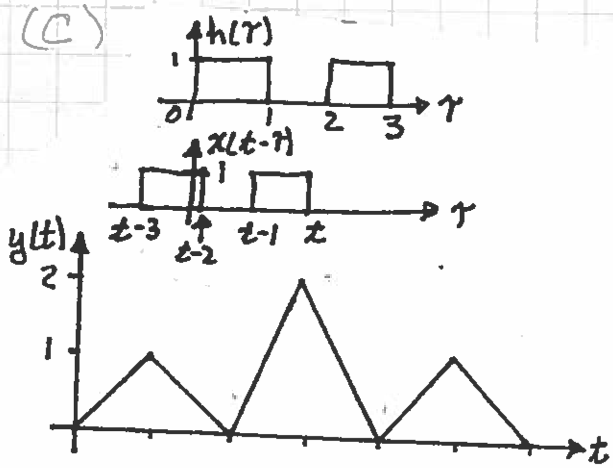
$y(t) = (t-t_0-t_1)\,u(t-t_0-t_1)$



Problem 3.5

(a) See solution for Problem 3.3 (b)
(b) See Solution for Problem 3.3 (c)

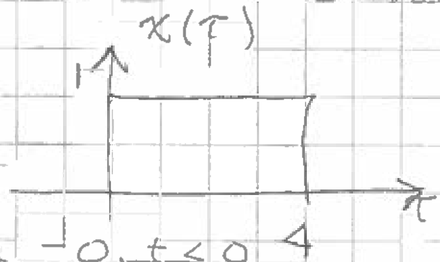
(c)

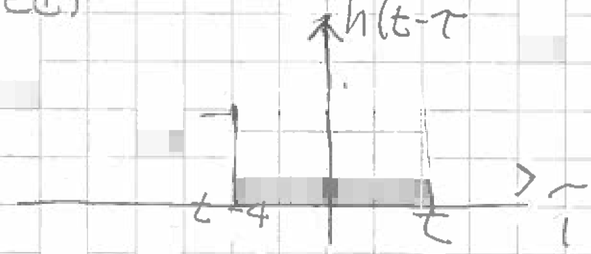


range	y(t)
$t<0$	0
$0<t<1$	t
$1<t<2$	$2-t$
$2<t<3$	$2(t-2)$
$3<t<4$	$2(4-t)$
$4<t<5$	$t-4$
$5<t<6$	$6-t$
$6<t$	0

PROBLEM 3.5 (continued)

d)



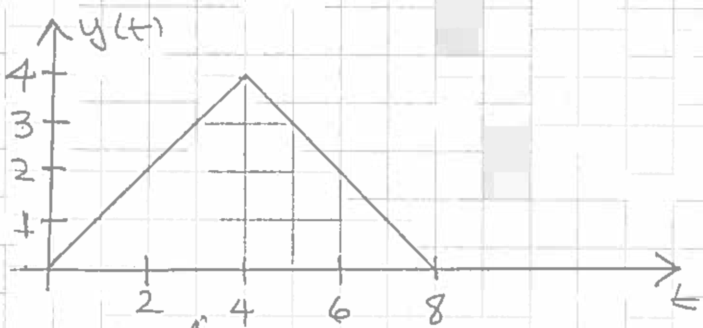


$y(t) = 0, \; t < 0$

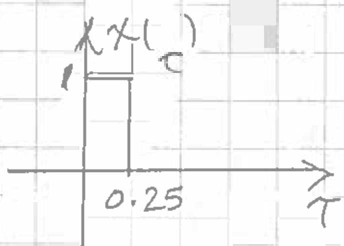
$0 < t < 4$: $\quad y(t) = \int_0^t d\tau = \tau \Big|_0^t = t, \quad 0 < t < 4$

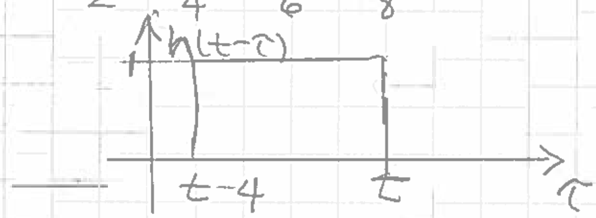
$4 < t < 8$: $\quad y(t) = \int_{t-4}^4 d\tau = \tau \Big|_{t-4}^4 = 4 - t + 4 = 8 - t, \quad 4 < t < 8$

$y(t) = 0, \; t > 8$



(e)





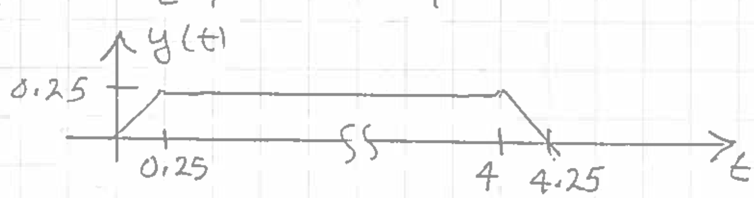
$y(t) = 0, \; t < 0$

$0 < t < 0.25$: $\quad y(t) = \int_0^t d\tau = \tau \Big|_0^t = t, \quad 0 < t < 0.25$

$0.25 < t < 4$: $\quad y(t) = \int_0^{0.25} d\tau = \tau \Big|_0^{0.25} = 0.25$

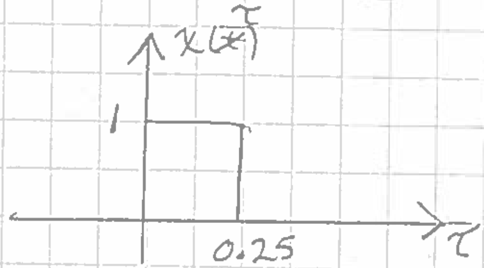
$4 < t < 4.25$: $\quad y(t) = \int_{t-4}^{0.25} d\tau = \tau \Big|_{t-4}^{0.25} = 0.25 - t + 4$

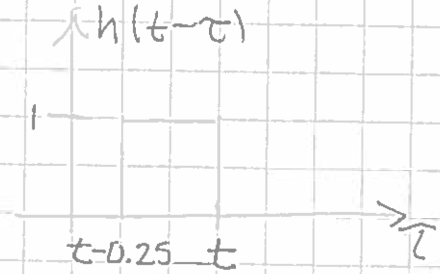
$y(t) = 0, \; t > 4.25$



PROBLEM 3.5 (Continued)

(f)



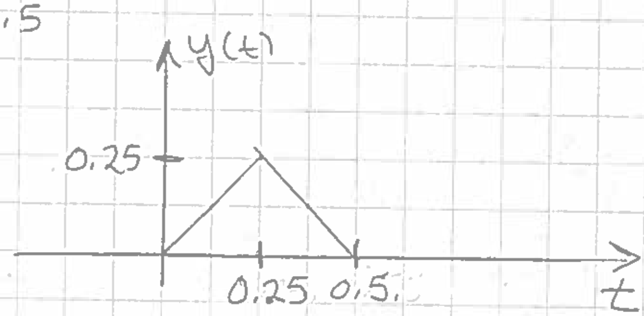


$y(t) = 0, \quad t < 0$

<u>$0 < t < 0.25$</u> : $y(t) = \int_0^t d\tau = \tau \Big|_0^t = t, \quad 0 < t < 0.25$

<u>$0.25 < t < 0.5$</u> : $y(t) = \int_{t-0.25}^{0.25} d\tau = \tau \Big|_{t-0.25}^{0.25} = 0.5 - t, \quad 0.25 < t < 0.5$

$y(t) = 0, \quad t > 0.5$



Problem 3.6

(a)

$t = 0$:

$h(\tau)x(-\tau) = 0$ for all τ, so $y(0) = \int_{-\infty}^{\infty} h(\tau)x(-\tau)d\tau = 0$.

$t = 1$:

$h(\tau)x(1-\tau) = -2(-2) = 4$ for $0 \le \tau < 1$
and $= 0$ elsewhere.
so $y(1) = \int_{-\infty}^{\infty} h(\tau)x(1-\tau)d\tau = \int_0^1 4d\tau = 4$.

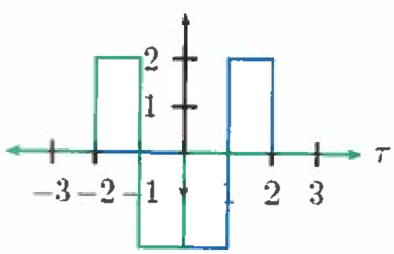
$t = 2$:

$h(\tau)x(2-\tau) = -2(2) = -4$ for $0 \le \tau < 2$
and $= 0$ elsewhere.
so $y(2) = \int_0^2 -4d\tau = -8$.

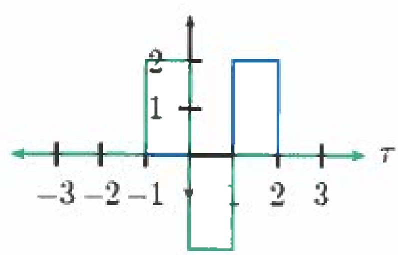
$t = 2.667$:

$h(\tau)x(2.667-\tau) = -2(2) = -4$ for $0.667 \le \tau < 1$,
$= 2(2) = 4$ for $1 \le \tau < 1.667$,
$= -4$ for $1.667 \le \tau < 2$,
and $= 0$ elsewhere.
Therefore $y(2.667) = (-4)(1-0.667) + 4(1.667-1) - 4(2-1.667) = -8(0.333) + 4(0.666) = 0$.

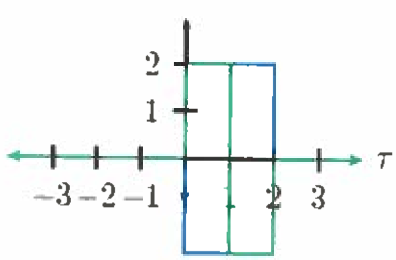
$h(\tau)$ (blue) and $x(-\tau)$ (green)



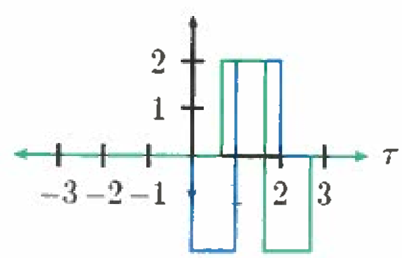
$h(\tau)$ (blue) and $x(1-\tau)$ (green)



$h(\tau)$ (blue) and $x(2-\tau)$ (green)



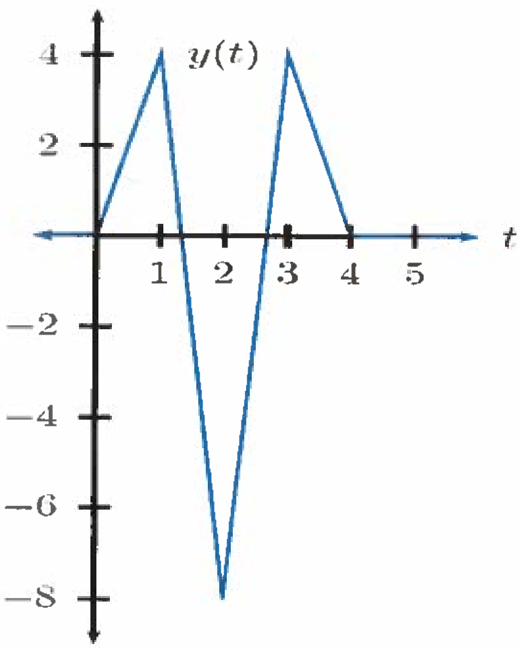
$h(\tau)$ (blue) and $x(2.667-\tau)$ (green)



PROBLEM 3.6 (Continued)

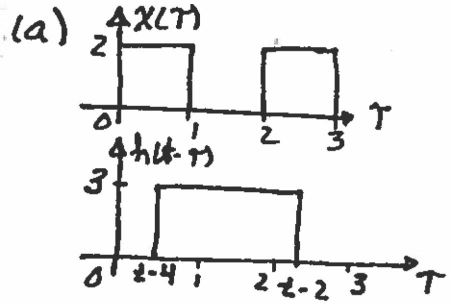
(b)

$$
\begin{aligned}
y(t) &= 0, t < 0 \\
&= \int_0^t -2(-2)d\tau = 4t, 0 \le t < 1 \\
&= \int_0^{t-1} 2(-2)d\tau + \int_{t-1}^{1} -2(-2)d\tau + \int_1^t -2(2) = -8(t-1) + 4(2-t) = -12t + 16, 1 \le t < 2 \\
&= \int_{t-2}^{1} 2(-2)d\tau + \int_1^{t-1} 2(2)d\tau + \int_{t-1}^{2} -2(2)d\tau = 12t - 32, 2 \le t < 3 \\
&= \int_{t-2}^{2} 2(2)d\tau = 4(4-t) = 16 - 4t, 3 \le t < 4 \\
&= 0, t \ge 4
\end{aligned}
$$



PROBLEM 3.7

(a)



$4 \le t \le 5,\ y(t) = \int_{t-4}^{1} (2)(3)\,d\tau + \int_{2}^{t-2} (2)(3)\,d\tau$

$= 6\tau\Big|_{t-4}^{1} + 6\tau\Big|_{2}^{t-2}$

$= 6 - 6t + 24 + 6t - 12 - 12 = \underline{6}$

(b) Maximum output $= \int_0^1 (2)(3)\,d\tau = 6\tau\Big|_0^1 = \underline{6}$

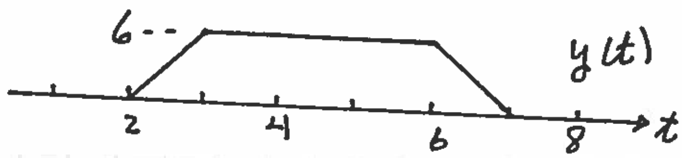
(c) Maximum output occurs $\begin{cases} \text{from } t-2=1 \Rightarrow t=3 \\ \text{to } t-4=2,\ t=6 \end{cases}$ $\therefore\ \underline{3 \le t \le 6}$

(d) $2 < t < 3,\ y(t) = \int_0^{t-2} (2)(3)\,d\tau = 6\tau\Big|_0^{t-2} = \underline{6(t-2)}$

$3 < t < 6,\ y(t) = \underline{6}$, from (a), (b), and (c),

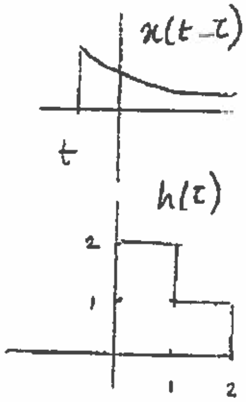
$6 < t < 7,\ y(t) = \int_{t-4}^{3} (2)(3)\,d\tau = 6\tau\Big|_{t-4}^{3} = 18 - 6t + 24 = \underline{42 - 6t}$

$t < 0$ and $t > 7,\ y(t) = 0$



PROBLEM 3.8

(a) $x(t) = e^{t} u(-t)$



① $t > 2$ no overlap $\therefore y(t) = 0$

② $1 \leq t \leq 2$ $\quad y(t) = \int_t^2 e^{t-\tau} d\tau = e^{t} \int_t^2 e^{-\tau} d\tau$

$$y(t) = e^{t}\left[e^{-t} - e^{-2}\right] = 1 - e^{t-2}$$

③ $0 \leq t \leq 1$, $\quad y(t) = 2\int_t^1 e^{t-\tau} d\tau + \int_1^2 e^{t-\tau} d\tau = 2(1 - e^{t-1})$

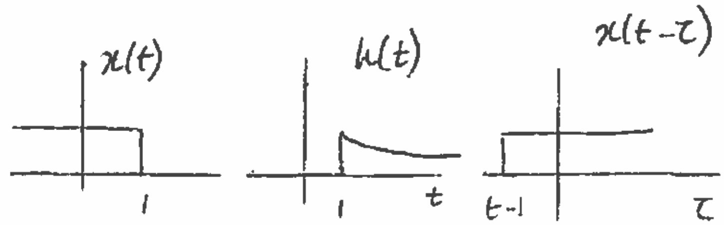
$$+ e^{t}(e^{-1} - e^{-2}) = 2 - e^{t-1} - e^{t-2}$$

④ $t < 0$, $\quad y(t) = 2\int_0^1 e^{t-\tau} d\tau + \int_1^2 e^{t-\tau} d\tau$

$$= 2(e^{t} - e^{t-1}) + e^{t}(e^{-1} - e^{-2}) = 2e^{t} - e^{t-1} - e^{t-2}$$

$$\therefore y(t) = (1 - e^{t-2})\left[u(t-1) - u(t-2)\right] + (2 - e^{t-1} - e^{t-2}) \times \left[u(t) - u(t-1)\right] + (2e^{t} - e^{t-1} - e^{t-2})\, u(-t)$$

(b)



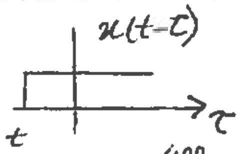
① $t-1 < 1$ or $t < 2$, $\quad y(t) = \int^{\infty} e^{-\tau} d\tau = e^{-1}$

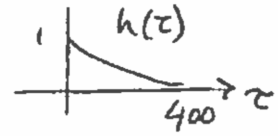
② $t-1 > 1$ or $t > 2$ $\quad y(t) = \int_{t-1}^{\infty} e^{-\tau} d\tau = -e^{-\tau}\Big|_{t-1}^{\infty} = e^{-(t-1)}$

$$\therefore y(t) = e^{-1} u(2-t) + e^{-(t-1)} u(t-2)$$

Problem 3.8 (continued)

(c) flip $x(t)$

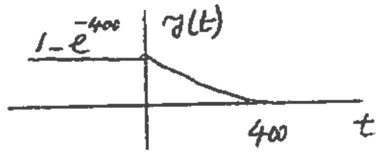




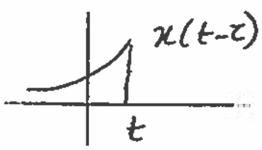
① $t<0$, $y(t) = \int_0^{400} e^{-\tau} d\tau = -e^{-\tau}\Big|_0^{400} = 1 - e^{-400}$

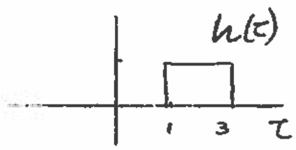
② $t>0$, $y(t) = \int_t^{400} e^{-\tau} d\tau = e^{-t} - e^{-400}$

③ $t>400$, $y(t) = 0$



(d)





① $t<1$, $y(t)=0$

② $1 \leq t \leq 3$, $y(t) = \int_1^t e^{-(t-\tau)} d\tau = e^{-t}\int_1^t e^{\tau} d\tau = e^{-t}(e^{t} - e)$
$= 1 - e^{-(t-1)}$

③ $t>3$, $y(t) = \int_1^3 e^{-(t-\tau)} d\tau = e^{-t}\int_1^3 e^{\tau} d\tau = e^{-t}(e^{3} - e)$
$= e^{-(t-3)} - e^{-(t-1)}$

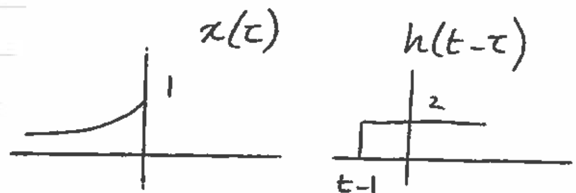
$\therefore y(t) = (1 - e^{-(t-1)})[u(t-1) - u(t-3)] +$
$(e^{-(t-3)} - e^{-(t-1)})\,u(t-3)$

PROBLEM 3.8 (continued)

(e)

$$
\begin{aligned}
y(t) &= e^{-at}[u(t)-u(t-2)] * u(t-2) \\
&= 0, t<2 \\
&= \int_0^{t-2} e^{-a\tau}\,d\tau = \tfrac{1}{a}(1-e^{-a(t-2)}), 2 \le t < 4 \\
&= \int_0^{2} e^{-a\tau}\,d\tau = \tfrac{1}{a}(1-e^{-a2}), t \ge 4 \\
&= \tfrac{1}{a}(1-e^{-a(t-2)})[u(t-2)-u(t-4)] + \tfrac{1}{a}(1-e^{-a2})u(t-4)
\end{aligned}
$$

(f)



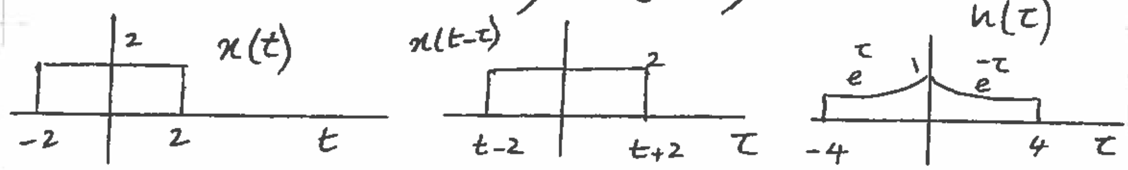
① $t-1<0$ $\quad y(t) = \int_{t-1}^{0} 2e^{\tau}\,d\tau = 2[1-e^{t-1}]$

② $t-1>0$ $\quad y(t) = 0$

$\therefore y(t) = 2[1-e^{-(1-t)}]u(1-t)$

PROBLEM 3.9

$$x_1(t) = 2u(t+2) - 2u(t-2)$$



① $t+2 < -4, \quad t < -6, \quad y(t) = 0$

② $-4 \leqslant t+2 \leqslant 0, \quad -6 \leqslant t \leqslant -2$

$$y(t) = \int_{-4}^{t+2} 2e^{\tau} d\tau = 2\left[e^{t+2} - e^{-4}\right]$$

③ $0 \leqslant t+2 \leqslant 4, \quad -2 \leqslant t \leqslant 2$

$$y(t) = 2\int_{t-2}^{0} e^{\tau} d\tau + 2\int_{0}^{t+2} e^{-\tau} d\tau = 2\left[1 - e^{t-2}\right] + 2\left[1 - e^{-(t+2)}\right]$$

④ $0 \leqslant t-2 \leqslant 4, \quad 2 \leqslant t \leqslant 6$

$$y(t) = \int_{t-2}^{4} e^{-\tau} d\tau = 2\left[e^{-(t-2)} - e^{-4}\right]$$

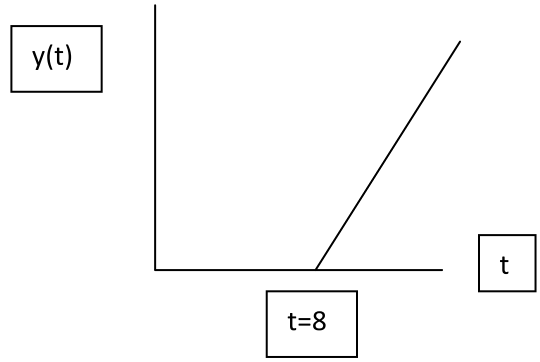
⑤ $t \geqslant 6, \quad y(t) = 0$

Problem 3.10

$$t < 8, y(t) = 0$$

$$t \geq 8, y(t) = \int_{8}^{t} d\tau = (t-8)$$

$$Therefore \quad y(t) = (t-8)u(t-8)$$



Problem 3.11

3.11

(a) $h(t) = h_1(t) * h_2(t) = \int_{-\infty}^{\infty} 2e^{-\tau} u(\tau)\, 2e^{-(t-\tau)} u(t-\tau)\, d\tau$

$= 4e^{-t} \int_0^t d\tau = 4t\, e^{-t} u(t)$

(b) $h(t) = \delta(t) * \delta(t) = \int_{-\infty}^{\infty} 2\delta(\tau) 2\delta(t-\tau) = \delta(t)$

(c) $h(t) = 2\delta(t-3) * 2\delta(t-3) = 4\,\delta(t-6)$

(d)

$(u(t-1) - u(t-5)) * (u(t-1) - u(t-5))$

$\Rightarrow \quad 0, \quad t < 2$

$\int_1^{t-1} 1(1)\, d\tau = t-2, \quad 2 \leq t < 6$

$\Rightarrow \quad \int_{t-5}^{5} 1(1)\, d\tau = 10 - t, \quad 6 \leq t < 10$

$\Rightarrow \quad 0, \quad t \geq 10$

$\Rightarrow \quad (t-2)[u(t-2) - u(t-6)]$

$+ (10-t)[u(t-6) - u(t-10)]$

PROBLEM 3.12

(a) $y_i(t)$ is the output of the ith system

$$y_1(t) = h_1(t) * x(t)$$

$$y_2(t) = h_2(t) * y_1(t) = h_1(t) * h_2(t) * x(t)$$

$$y_3(t) = h_1(t) * h_3(t) * x(t)$$

$$y_5(t) = h_5(t) * x(t)$$

$$y(t) = y_2(t) + y_4(t) = y_2(t) + [y_3(t) + y_5(t)] * h_4(t)$$

$$y(t) = x(t) * [h_1(t) * h_2(t) + h_1(t) * h_3(t) * h_4(t) + h_4(t) * h_5(t)]$$

$$h(t) = h_1(t) * h_2(t) + h_1(t) * h_3(t) * h_4(t) + h_4(t) * h_5(t)$$

(b) $h(t) = u(t) * 5\delta(t) + u(t) * 5\delta(t) * u(t) + u(t) * e^{-2t}u(t)$

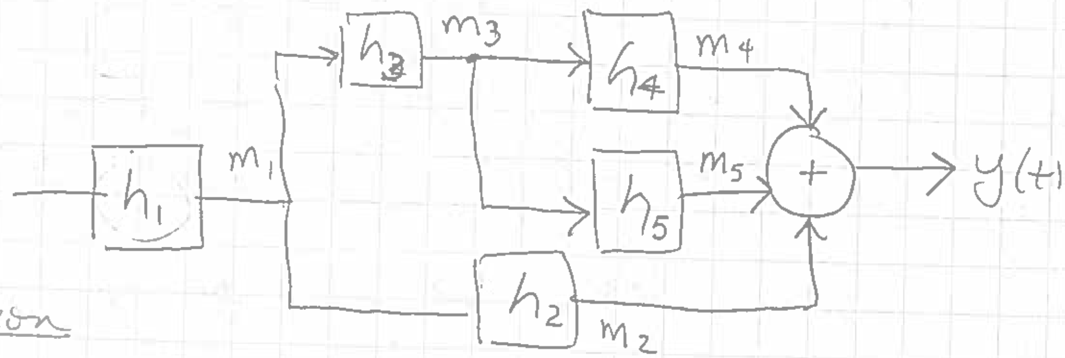
now $u(t) * e^{-2t}u(t) = \int_{-\infty}^{\infty} u(\tau) e^{-2(t-\tau)} u(t-\tau) d\tau$

$$= \int_0^t e^{-2(t-\tau)} d\tau = e^{-2t} \int_0^t e^{2\tau} d\tau = \tfrac{1}{2}(1 - e^{-2t}) u(t)$$

$$\therefore h(t) = 5u(t) + 5t\,u(t) + \tfrac{1}{2}(1 - e^{-2t}) u(t)$$

PROBLEM 3.13

$$h_1 = h_3 = 2\delta(t)$$
$$h_2 = h_4 = u(t)$$
$$h_5 = 2u(t)$$



Solution

(a) $y = m_2 + m_4 + m_5$

$m_1 = x * h_1$, $\boxed{m_2 = x * h_1 * h_2}$ ($m_1 * h_2$)

$m_3 = m_1 * h_3 = x * h_1 * h_3$

$m_4 = m_3 * h_4 = \boxed{x * h_1 * h_3 * h_4}$

$m_5 = m_3 * h_5 = \boxed{x * h_1 * h_3 * h_5}$

$\therefore y = x * h_1 * h_2 + x * h_1 * h_3 * h_4 + x * h_1 * h_3 * h_5$

$y = x * h_1 * [h_2 + h_3 * (h_4 + h_5)] = h(t) * x(t)$

$h(t) = h_1(t) * h_2(t) + h_1(t) * h_3(t) * h_4(t) + h_1(t) * h_3(t) * h_5(t)$

(b) impulse response $\Rightarrow x(t) = \delta(t)$, $y(t) = h(t)$

$h(t) = \delta(t) * 2\delta(t) * [u(t) + 2\delta(t) * (u(t) + 2u(t))]$

$= \delta(t) * 2\delta(t) * [u(t) + 2\delta(t) * 3u(t)]$

$h(t) = 2\delta(t) * [u(t) + 6u(t)]$

$= 2u(t) + 12u(t)$

$h(t) = 14u(t)$

PROBLEM 3.13 (Continued)

(c) BLOCK 1 = BLOCK 3 = amplifier with gain = 2
BLOCK 2 = BLOCK 4 = unity-gain integrator
BLOCK 5 = integrating amplifier with gain = 2

(d) $m_1 = 2\delta(t)$, $m_2 = 2\delta(t) * u(t) = 2u(t)$

$m_3 = 2\delta(t) * 2\delta(t) = 4\delta(t)$

$m_4 = 4\delta(t) * u(t) = 4u(t)$

$m_5 = 4\delta(t) * 2u(t) = 8u(t)$

$$h(t) = m_2 + m_4 + m_5 = 14u(t)$$

(e) $\delta(t)h(t) = 14u(t)$ ✓

PROBLEM 3.14

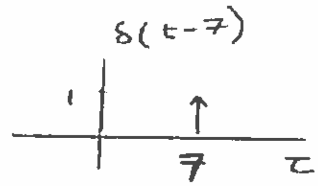
a) $x(t) = \delta(t) \longrightarrow y(t) = h(t)$

$y(t) = x(t-7)$

$h(t) = \delta(t-7)$

b) $y(t) = \int_{-\infty}^{t} x(\tau - 7)\, d\tau$

$h(t) = \int_{-\infty}^{t} \delta(\tau - 7)\, d\tau$

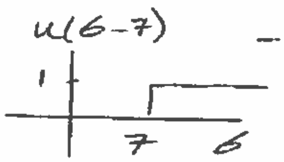


$t < 7, \quad h(t) = 0$

$t > 7, \quad h(t) = 1 \qquad \therefore h(t) = u(t-7)$

c) $y(t) = \int_{-\infty}^{t} \left[\int_{-\infty}^{\sigma} x(\tau - 7)\, d\tau \right] d\sigma$ let $x(t) = \delta(t)$

$h(t) = \int_{-\infty}^{t} \left[\int_{-\infty}^{\sigma} \delta(\tau - 7)\, d\tau \right] d\sigma = \int_{-\infty}^{t} u(\sigma - 7)\, d\sigma$



$t < 7, \quad h(t) = 0$

$t \geq 7, \quad h(t) = \int_{7}^{t} d\sigma = (t-7)$

$\therefore h(t) = (t-7)\, u(t-7)$

PROBLEM 3.15

let $x(t-\tau) = \begin{cases} 1 & h(\tau) > 0 \\ -1 & h(\tau) < 0 \end{cases}$ $\therefore x$ is bounded

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} h(\tau)\,x(t-\tau)\,d\tau$$

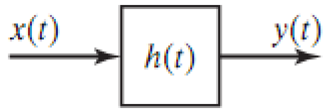
$$h(\tau)\,x(t-\tau) = \begin{cases} h(\tau), & h(\tau) > 0 \\ -h(\tau), & h(\tau) < 0 \end{cases}$$

$$\therefore h(\tau)\,x(t-\tau) = |h(\tau)|$$

$\therefore y(t) = \int_{-\infty}^{\infty} |h(\tau)|\,d\tau$ which is assumed unbounded

$\therefore$ system is not BIBO stable

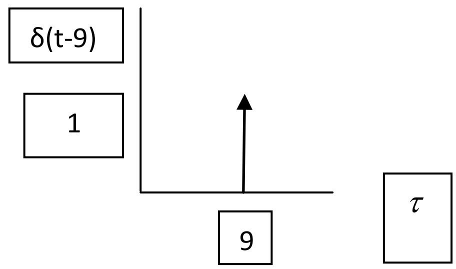
Problem 3.16 .



a)x(t)=δ(t),y(t)=h(t), so y(t)=x(t-9),h(t)=δ(t-9)

b) $y(t)=\int_{-\infty}^{t} x(\tau-9)d\tau$,

$h(t)=\int_{-\infty}^{t} \delta(\tau-9)d\tau$,



if t<9,h(t)=0

if t>9, h(t)=1

Therefore h(t)=u(t-9)

c) $y(t)=\int_{-\infty}^{t}\left[\int_{-\infty}^{\sigma} x(\tau-9)d\tau\right]d\sigma$ let x(t)=δ(t)

$$h(t)=\int_{-\infty}^{t}\left[\int_{-\infty}^{\sigma} x(\tau-9)d\tau\right]d\sigma=\int_{-\infty}^{\sigma} u(\tau-9)d\tau$$

t<9, h(t)=0

t>9, h(t)= $\int_{9}^{t} d\sigma=(t-9)$

Therefore h(t)=(t-7)u(t-7)

d) x(t)=δ(t),y(t)=h(t), so y(t)=x(t+9),h(t)=δ(t+9)

Problem 3.17 Linear

a) Not time invariant

b) δ(t)=sin(4t)δ(t)=1 δ(t)= δ(t)

c) δ(t-π/2)=sin4t δ(t-π/2)=sin(π/2) δ(t-π/2)=1 δ(t)=1

Problem 3.18

a) Stable,casual
b) Not Stable,casual
c) Stable, not casual
d) Not Stable, not casual
e) Stable, not casual
f) Stable, casual
g) Stable, casual
h) Stable, casual

PROBLEM 3.19

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau)\, d\tau$$

(a) $h(t) = \int_{-\infty}^{t} e^{-(t-\tau)} \delta(\tau)\, d\tau = \begin{cases} e^{-t}, & t \geq 0 \\ 0, & t < 0 \end{cases} = \underline{e^{-t} u(t)}$

(b) yes, $h(t) = 0,\ t < 0$

(c) $y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} u(\tau+1)\, d\tau = \int_{-1}^{t} e^{-t} e^{\tau}\, d\tau$

$$= e^{-t}\left(e^{\tau}\Big|_{-1}^{t}\right) = e^{-t}(e^{t} - e^{-1})\, u(t+1)$$

$$= \underline{[1 - e^{-(t+1)}]\, u(t+1)}$$

(d) $y(t) = h(t) * \delta(t) - h(t) * \delta(t-1) * \delta(t)$

$$= h(t) * \delta(t) - h(t-1) * \delta(t) = h(t) - h(t-1)$$

$$= \underline{e^{-t} u(t) - e^{-(t-1)} u(t-1)}$$

(e) (i) $y(t) = y_c(t) - y_c(t)\Big|_{t+1 \leftarrow t}$

$$= \underline{[1 - e^{-(t+1)}]\, u(t+1) - [1 - e^{-t}]\, u(t)}$$

(ii) $y(t) = h(t) * u(t+1) = \int_{-\infty}^{\infty} u(t-\tau+1)\left[e^{-\tau} u(\tau) - e^{-(\tau-1)} u(\tau-1)\right] d\tau$

$$= \int_{0}^{\infty} e^{-\tau} u(t+1-\tau)\, d\tau - e^{1} \int_{1}^{\infty} e^{-\tau} u(t+1-\tau)\, d\tau = I_1 - I_2$$

$I_1 = \int_{0}^{t+1} e^{-\tau}\, d\tau = -e^{-\tau}\Big|_{0}^{t+1} = [1 - e^{-(t+1)}]\, u(t+1)$

$I_2 = e^{1} \int_{1}^{t+1} e^{-\tau}\, d\tau = e^{1}(-e^{-\tau})\Big|_{1}^{t+1} = e^{1}(e^{-1} - e^{-(t+1)})\, u(t)$

$$= (1 - e^{-t})\, u(t)$$

$\therefore\ y(t) = \underline{[1 - e^{-(t+1)}]\, u(t+1) - [1 - e^{-t}]\, u(t)}$

3.20.

3.20

(a) $y(t) = \int_{-\infty}^{t} e^{-4(t-\tau)} x(\tau-1)\, d\tau$

(i) $h(t) = \int_{-\infty}^{t} e^{-4(t-\tau)} \delta(\tau-1)\, d\tau$

$= e^{-4(t-1)} u(t-1)$

(ii) $h(t) = 0$ for $t < 0$ ∴ causal

(iii) $\int_{-\infty}^{\infty} \left| e^{-4(t-1)} u(t-1) \right| dt = \int_{1}^{\infty} e^{-4(t-1)} dt$

$= e^{4} \left(\frac{e^{-4t}}{-4} \right) \Big|_{1}^{\infty} = e^{4} \left(\frac{e^{-4}}{4} \right) = 1/4$

= stable

(b) (i) $y(t) = \int_{-\infty}^{t} e^{-4(t+\tau)} x(\tau+1)\, d\tau$

(i) $h(t) = \int_{-\infty}^{t} e^{-4(t+\tau)} \delta(\tau+1)\, d\tau = e^{-4(t+1)} u(t+1)$

(ii) $h(t) \neq 0$ for $t<0$ ∴ non causal

(iii) $\int_{-\infty}^{\infty} \left| e^{-4(t+1)} u(t+1) \right| dt = \int_{-1}^{\infty} e^{-4(t+1)} dt$

$= e^{4} \left(\frac{e^{-4t}}{-4} \right) \Big|_{-1}^{\infty} = \frac{e^{8}}{4}$ = stable

3.20

(c) $y(t) = \int_{-\infty}^{\infty} e^{-2(t-\tau)}\, x(\tau-1)\, d\tau$

(i) $h(t) = \int_{-\infty}^{\infty} e^{-2(t-\tau)}\, \delta(\tau-1)\, d\tau = e^{-2(t-1)}$

(ii) $h(t) \neq 0, \quad t<0 \quad \therefore$ non causal

(iii) $\int_{-\infty}^{\infty} \left| e^{-2(t-1)} \right| dt = \int_{-\infty}^{\infty} e^{-2t} e^{2}\, dt$

$$= e^{2} \left. \frac{\left(e^{-2t}\right)}{-2} \right|_{-\infty}^{\infty}$$

unbounded $\therefore$ unstable

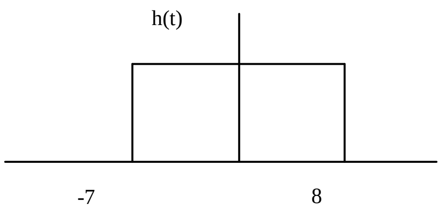
Problem 3.21

a)Not casual

b) Stable

c) $x(t) = \delta(t-2) - 2\delta(t+2)$

y(t)=x(t)*h(t)=h(t-2)-2h(t+2)



Problem 3.22

(a) Non casual

(b) Stable

(c)

$$y(t) = h(t) * \delta(t-3) - 2h(t) * \delta(t-5) = h(t-3) - 2h(t-5)$$

$$= \left[u(t-3) - 2u(t+1) + u(t-5)\right] - 2\left[u(t-5) - 2u(t-1) + u(t-7)\right]$$

Problem 3.23.

(a) $(t-1-2)u(t-1-2)=(t-3)u(t-3)$

(b) x(t)=1 if t>1,h(t) =1 if t>2

Problem 3.24

a) Casual since h(t)=0 when t<0
b) Stable
c) Not casual and not stable

PROBLEM 3.25

(i) Characteristic equation: $s + 3 = 0$, solution $s = -3$

$\Longrightarrow y_c(t) = Ce^{-3t}u(t)$

Forced response of the form: $y_p(t) = Pu(t)$ where $\frac{dy_p(t)}{dt} + 3y_p(t) = 3u(t)$

$\Longrightarrow 0 + 3Pu(t) = 3u(t) \Longrightarrow P = 1$

$y(t) = y_c(t) + y_p(t) = (Ce^{-3t} + 1)u(t)$

Need $y(0) = C + 1 = -1 \Longrightarrow C = -2$

$\Longrightarrow y(t) = (-2e^{-3t} + 1)u(t)$

This clearly satisfies the differential equation and initial conditions because

$\frac{dy(t)}{dt} + 3y(t) = 6e^{-3t} + 3(-2e^{-3t} + 1) = 3, t > 0$

$y(0) = -2e^{-3 \cdot 0} + 1 = -1$

(ii) Characteristic equation: $s + 3 = 0$, solution $s = -3$

$\Longrightarrow y_c(t) = Ce^{-3t}u(t)$

Forced response of the form $y_p(t) = Pe^{-2t}u(t)$ where $\frac{dy_p(t)}{dt} + 3y_p(t) = 3e^{-2t}u(t)$

$\Longrightarrow (-2P + 3P)e^{-2t}u(t) = 3e^{-2t}u(t) \Longrightarrow P = 3$

$y(t) = y_c(t) + y_p(t) = (Ce^{-3t} + 3e^{-2t})u(t)$

Need $y(0) = C + 3 = 2 \Longrightarrow C = -1$

$\Longrightarrow y(t) = (3e^{-2t} - e^{-3t})u(t)$

This clearly satisfies the differential equation and initial conditions since

$\frac{dy(t)}{dt} + 3y(t) = (-6e^{-2t} + 3e^{-3t}) + 3(3e^{-2t} - e^{-3t}) = 3e^{-2t}, t > 0$

$y(0) = 3e^{-2 \cdot 0} - e^{-3 \cdot 0} = 2$

(iii) Characteristic equation: $S + 10 = 0$

$\Rightarrow y_c(t) = Ce^{-10t}$

$y_p(t) = Pe^{-t} \Rightarrow -Pe^{-t} + 10e^{-t} = e^{-t} \Rightarrow P = 1/9$

$y(t) = \frac{1}{9}e^{-t} + Ce^{-10t}, \quad t \geq 0$

$y(0) = \frac{1}{9} + C = 2 \Rightarrow C = 2 - 1/9 = \frac{17}{9}$

$\therefore y(t) = \frac{1}{9}e^{-t} + \frac{17}{9}e^{-10t} \Rightarrow y(0) = 2$ ✓

$\frac{dy(t)}{dt} = -\frac{1}{9}e^{-t} - \frac{170}{9}e^{-10t}$

$\therefore -\frac{1}{9}e^{-t} - \frac{170}{9}e^{-10t} + \frac{10}{9}e^{-t} + \frac{170}{9}e^{-10t} = e^{-t}$ ✓

PROBLEM 3.25 (Continued)

(iv) Characteristic equation: $s^2+6s+5=0$

$(s+5)(s+1)=0 \Rightarrow \underline{y_c(t) = C_1e^{-t}+C_2e^{-5t}}$

$y_p(t) = P \Rightarrow 0+6(0)+5P=10 \Rightarrow \underline{P=2}$

$\therefore \underline{y(t) = 2 + C_1e^{-t}+C_2e^{-5t}}$

$y(0) = 2+C_1+C_2 = -1 \Rightarrow C_1+C_2 = -3.$

$\left.\frac{dy(t)}{dt}\right|_{t=0} = -C_1-5C_2 = 0 \Rightarrow C_1 = -5C_2$

$-5C_2+C_2 = -3 \Rightarrow C_2 = 3/4$

$C_1 = -\frac{15}{4}$

$y(t) = 2 - \frac{15}{4}e^{-t}+\frac{3}{4}e^{-5t}$

$y(0) = 2 - 15/4 + 3/4 = -1$ ✓

$\frac{dy(t)}{dt} = \frac{15}{4}e^{-t} - \frac{15}{4}e^{-5t} \Rightarrow \left.\frac{dy(t)}{dt}\right|_{t=0} = 0$ ✓

(v) Characteristic equation: $-0.7s+1=0 \implies s-\frac{1}{0.7}=0$, solution $s=\frac{1}{0.7}=10/7$

$\implies y_c(t) = Ce^{t/0.7}u(t)$

Forced response of the form $y_p(t) = Pe^{3t}u(t)$ where $\frac{dy_p(t)}{dt} - \frac{10}{7}y_p(t) = \frac{-30}{7}e^{3t}u(t)$

$\implies 3P - \frac{10}{7}P = \frac{-30}{7}$ Solving for P gives $P = -\frac{30}{11}$

$y(t) = y_c(t)+y_p(t) = (Ce^{t/0.7} - (\frac{30}{11})e^{3t})u(t)$

Need $y(0) = C - \frac{30}{11} = -1 \implies C = \frac{19}{11}$

$\implies y(t) = (\frac{19}{11}e^{t/0.7} - \frac{30}{11}e^{3t})u(t)$

This clearly satisfies the differential equation and initial conditions since

$\frac{dy(t)}{dt} - \frac{1}{0.7}y(t) = 2.47e^{t/0.7} - 3(\frac{30}{11})e^{3t} - 2.47e^{t/0.7} + \frac{300}{77}e^{3t} = \frac{-30}{7}e^{3t}$

$y(0) = \frac{19}{11} - \frac{30}{11} = -1$

PROBLEM 3.25 (continued)

(vi) Characteristic equation: $-10S + 10S = 0$

$\Rightarrow S = 1$

$y_c(t) = C_1 e^{t}$, $y_P(t) = P_1 \cos(t) + P_2 \sin(t)$

$+10[P_1 \sin(t) - P_2 \cos(t)] + 10[P_1 \cos(t) + P_2 \sin(t)]$

$= 20 \cos t, \quad t \geq 0$

$\Rightarrow 10P_1 + 10P_2 = 0 \quad \Rightarrow P_1 + P_2 = 0$

$-10P_2 + 10P_1 = 20 \quad \Rightarrow P_1 - P_2 = 2$

$\Rightarrow \underline{P_1 = 1, P_2 = -1}$

$\therefore y(t) = C_1 e^{t} + \cos(t) - \sin(t)$

$y(0) = C_1 + 1 = -10 \Rightarrow C_1 = -11$

$y(t) = -11e^{t} + \cos t - \sin(t) \quad \Rightarrow \underline{y(0) = -10}$ ✓

$\frac{dy(t)}{dt} = -11e^{t} - \sin(t) - \cos(t)$

$\therefore \cancel{-110e^{t}} + \cancel{10\sin(t)} + 10\cos(t) + \cancel{110e^{t}} + 10\cos(t)$

$\cancel{-10\sin(t)} = 20\cos(t), \; t \geq 0$ ✓

(vii) Characteristic Equation: $S^2 + S + 2 = 0 \Rightarrow S_{1,2} = -\frac{1}{2} \pm j\frac{\sqrt{7}}{2}$

$y_c(t) = C_1 e^{-t/2} e^{j\frac{\sqrt{7}}{2}t} + C_2 e^{-t/2} e^{-j\frac{\sqrt{7}}{2}t}$

$y_P(t) = P \quad \Rightarrow 2P = 3 \quad \Rightarrow \underline{P = 3/2}$

$y(t) = 3/2 + C_1 e^{-(1-j\sqrt{7})t/2} + C_2 e^{-(1+j\sqrt{7})t/2}$

$y(0) = 3/2 + C_1 + C_2 = 2 \quad \Rightarrow \underline{C_1 + C_2 = \frac{1}{2}}$

$\left.\frac{dy(t)}{dt}\right| = -\frac{1}{2}(1-j\sqrt{7})C_1 - \frac{1}{2}(1+j\sqrt{7})C_2 = 1$

$+\frac{1}{2}(1-j\sqrt{7})C_1 + 1/2(1-j\sqrt{7})C_2 = +1/4(1-j\sqrt{7})$

$-j\sqrt{7}\,C_2 = \frac{5}{4} - j\frac{\sqrt{7}}{4}$

$C_2 = \frac{1}{4} + j\frac{5}{4\sqrt{7}} \quad \Rightarrow C_1 = \frac{1}{2} - C_2 = \frac{1}{4} - j\frac{5}{4\sqrt{7}}$

PROBLEM 3.25(vii) (Continued)

$$y(t) = \tfrac{3}{2} + \left(\frac{1}{4} - j\frac{5}{4\sqrt{7}}\right)e^{-\frac{1}{2}(1-j\sqrt{7})t} + \left(\frac{1}{4} + j\frac{5}{4\sqrt{7}}\right)e^{-\frac{1}{2}(1+j\sqrt{7})t}$$

$$y(0) = \frac{3}{2} + \frac{1}{4} - j\frac{5}{4\sqrt{7}} + \frac{1}{4} + j\frac{5}{4\sqrt{7}} = 2 ✓$$

$$\left.\frac{dy(t)}{dt}\right|_{t=0} = \left(\frac{1}{4} - j\frac{5}{4\sqrt{7}}\right)\left(-\frac{1}{2} + j\frac{\sqrt{7}}{2}\right) + \left(\frac{1}{4} + j\frac{5}{4\sqrt{7}}\right)\left(-\frac{1}{2} - j\frac{\sqrt{7}}{2}\right)$$

$$= -\frac{1}{8} \cancel{+ j\frac{\sqrt{7}}{8}} \cancel{+ j\frac{5}{8\sqrt{7}}} + \frac{5}{8} - \frac{1}{8} \cancel{- j\frac{\sqrt{7}}{8}} \cancel{- j\frac{5}{8\sqrt{7}}} + \frac{5}{8} = 1 ✓$$

PROBLEM 3.26

$$y(t) = h_1(t) * [x(t) - h_2(t) * y(t)]$$
$$= h_1(t) * x(t) - h_1(t) * h_2(t) * y(t)$$

$$h_1(t) = 2u(t) \quad , \quad h_2(t) = \tfrac{1}{2}\delta(t)$$

$$y(t) = x(t) * 2u(t) - \tfrac{1}{2}\delta(t) * 2u(t) * y(t)$$

$$\tfrac{1}{2}\delta(t) * 2u(t) = u(t)$$

$$\therefore \; y(t) = x(t) * 2u(t) - y(t) * u(t)$$
$$= 2\int_{-\infty}^{\infty} x(\tau)u(t-\tau)d\tau - \int_{-\infty}^{\infty} y(\tau)u(t-\tau)d\tau$$

$$y(t) = 2\int_{-\infty}^{t} x(\tau)d\tau - \int_{-\infty}^{t} y(\tau)d\tau$$

$$\frac{dy(t)}{dt} = 2x(t) - y(t)$$

$$\Rightarrow \boxed{\frac{dy(t)}{dt} + y(t) = 2x(t)}$$

PROBLEM 3.27

(a) characteristic equation is $s^2 - 2.5s + 1 = (s-2)(s-0.5) = 0$; roots are $s = 2, 0.5$; modes are $e^{2t}, e^{0.5t}$. Unstable since roots > 0, and real.

(b) characteristic equation $s^2 + 9 = (s-3j)(s+3j) = 0$, roots $s = 3j, -3j$; modes e^{3jt}, e^{-3jt}. Unstable since real part of roots is 0 (roots lie on imaginary axis).

(c) Characteristic equation: $S^2 + 3.5S - 2 = 0$
roots are $S_1 = -4$, $S_2 = 0.5$
modes are e^{-4t}, $e^{0.5t}$
unstable, $\lim_{t\to\infty} e^{0.5t} \to \infty$

(d) Characteristic equation: $S^3 + 5S^2 + 4S + 3 = 0$
roots are $S_1 = -4.22$, $S_2 = -0.3897 + j0.7476$
$S_3 = -0.3897 - j0.7476$
modes: $e^{-4.22t}$, $e^{(-0.3897+j0.7476)t}$, $e^{(-0.3897-j0.7476)t}$
Stable since all roots have negative real parts.

(e) Characteristic equation: $S^3 + 2S^2 + 4S + 8 = 0$
roots: -2, $+j2$, $-j2$
Modes: e^{-2t}, e^{j2t}, e^{-j2t}
not stable, 2 root do not have negative real parts.

(f) Characteristic equation: $S^3 + 2S^2 + 4S + 16 = 0$
roots: -2.7064, $0.3532 + j2.4056$, $0.3532 - j2.4056$
Modes: $e^{-2.7064t}$, $e^{(0.353t + j2.4056)t}$, $e^{(0.3532 - j2.4056)t}$
Unstable — 2 root have positive real parts.

PROBLEM 3.28

(a) Stable. All characteristic roots have negative real parts

(b) characteristic equation: $s^2 + 1.5s - 1 = 0$
roots: $s_1 = -2$, $s_2 = +0.5$

unstable, 1 root is positive and real.

(c) characteristic equation: $s^2 + 2s = 0$
roots: $s = 0$, $s = -2$

unstable: one root has non-negative real part.

(d) characteristic equation: $s^3 + 2s^2 + 8s + 32 = 0$
roots: -2.9559, $0.4780 + j3.2553$, $0.4780 - j3.2553$

unstable: 2 root have positive real parts.

(e) characteristic equation: $s^3 + 2s^2 + 8s + 16 = 0$
roots: -2, $j2.8284$, $-j2.8284$

unstable. 2 root have non-negative real parts.

(f) characteristic equation: $s^2 + 2s^2 + 8s + 8$
roots: -2, $j2$, $-j2$

unstable. 2 roots have non-negative real parts

PROBLEM 3.29

(a) (i) From solution to Problem 3.25, $s=-3$.

$\therefore$ mode is e^{-3t}.

(ii) mode is e^{-3t}.

(iii) $s=-10$, $\therefore$ mode is e^{-10t}

(iv) $s_1=-1$, $s_2=-5$, $\therefore$ Mode are e^{-t}, e^{-5t}

(v) $s=\frac{10}{7}$, mode is $e^{10/7 t}$

(b) $e^{-at}=e^{-t/\tau} \Rightarrow \tau=\frac{1}{a}$ = time constant.

(i) $e^{-3t}=e^{-t/\tau} \Rightarrow \tau=1/3\,(s)$.

(ii) $e^{-3}=e^{-t/\tau} \Rightarrow \tau=1/3\,(s)$.

(iii) $e^{-10t}=e^{-t/\tau} \Rightarrow \tau=1/10\,(s)$

(iv) $e^{-5t}=e^{-t/\tau} \Rightarrow \tau_2=\frac{1}{5}\,(s)$

$e^{-t}=e^{-t/\tau} \Rightarrow \tau_1=1\,(s)$

(v) $e^{+10/7 t}=e^{-t/\tau} \Rightarrow \tau=-\frac{7}{10}\,(s)$ ← unstable mode

(c) The natural response of a stable system reaches steady-state in approximately four time constants

$T_s \simeq 4\tau$

(i) $\tau=1/3\,(s) \Rightarrow T_s \simeq 4/3\,(s)$

(ii) $T_s \simeq 4/3\,(s)$

(iii) $\tau=\frac{1}{10}\,(s) \Rightarrow T_s \simeq 4/10\ s$

(iv) $\tau_1=1\,(s)$, $\tau_2=\frac{1}{5}\,(s)$ $T_s \simeq 4\tau_1=4\,s$

(v) the system unstable – it will not reach steady-state.

(d) Char. eqn: $s^2+9=0 \Rightarrow s_1=j3$, $s_2=-j3$

modes: e^{j3t}, e^{-j3t}

the real part of each root is zero, therefore the time constant is undefined. The natural response is oscillatory.

PROBLEM 3.30

(a) Characteristic equation: $0.01s^2 + 1 = 0$

$s^2 + 100 = 0 \Rightarrow s_1 = j10,\ s_2 = -j10$

modes: $e^{j10},\ e^{-j10}$

(b) $y_c(t) = Ce^{j\theta}e^{j10t} + Ce^{-j\theta}e^{-j10t}$

$= 2C\left[\dfrac{e^{j(10t+\theta)} + e^{-j(10t+\theta)}}{2}\right]$

from Euler's relation: $\cos(\phi) = \dfrac{e^{j\phi} + e^{-j\phi}}{2}$

$y_c(t) = 2C\cos(10t + \theta)$

(c) The diffential equation of the system is

$\dfrac{d^2y(t)}{dt} + 100y(t) = 100x(t)$, $x(t) = e^{-t}u(t)$

$y_p(t) = Pe^{-t}$

$Pe^{-t} + 100Pe^{-t} = 100e^{-t}$, $t \geq 0$

$P = \dfrac{100}{101}$

$y(t) = \dfrac{100}{101}e^{-t} + 2C\cos(10t + \theta)$

$y(0) = 0 = \dfrac{100}{101} + 2C\cos\theta$

$\Rightarrow C\cos\theta = -\dfrac{50}{101}$

$\left.\dfrac{dy(t)}{dt}\right|_{t=0} = -\dfrac{100}{101} - 20C\sin\theta \ \ +t\ \ = 0$

$\Rightarrow C\sin\theta = -\dfrac{5}{101}$

$\dfrac{C\sin\theta}{C\cos\theta} = \tan\theta = 0.1 \Rightarrow \theta = \tan^{-1}(0.1) = 5.71$ (rad)

$\cos\theta = 0.995$, $\sin\theta = 0.0995$

$C = \dfrac{-50}{101} \cdot \dfrac{1}{0.995} = -0.4975$

$y(t) = \dfrac{100}{101}e^{-t} - 0.9951\cos(10t + 327.16°)$

PROBLEM 3.30 (Continued)

(d) $y(0) = \frac{100}{101} + 0.9951 \cos(327.16°) \simeq 0$ ✓

$$\left.\frac{dy(t)}{dt}\right|_{t=0} = \frac{-100}{101} + (0.995)(10)\sin(327.16°) \simeq 0$$ ✓

PROBLEM 3.31

(a) $x(t) = 3u(t) = 3e^{0t} \Rightarrow s = 0$

(i) $H(0) = 10/4 = 2.5$

$y_{ss}(t) = H(0)x(t) = (3)(2.5) = \underline{7.5}$

(ii) $H(0) = \frac{5}{10} = 0.5 \Rightarrow y_{ss}(t) = (0.5)(3) = \underline{1.5}$

(b) $x(t) = 3e^{4t}u(t) \Rightarrow s = 4$

(i) $H(4) = \frac{10}{4+4} = 1.25$

$y_{ss}(t) = H(4)x(t) = (1.25)(3e^{4t}) = \underline{3.75e^{4t}}$

(ii) $H(4) = \frac{2(4)+5}{4^2+2(4)+10} = \frac{13}{34} \Rightarrow y_{ss}(t) = \underline{\frac{39}{34}e^{4t}}$

(c) $x(t) = 3\cos 4t, \Rightarrow s_{1,2} = \pm j4$

(i) $H(j4) = \frac{10}{j4+4} = 1.768\angle -45°$

$y_{ss}(t) = H(j4)x(t) = (1.768)(3)\cos(4t-45°)$

$= \underline{5.303\cos(4t-45°)}$

(ii) $H(j4) = \frac{5+j8}{-16+j8+10} = \frac{5+j8}{-6+j8} = 0.943\angle -68.9°$

$y(t) = (0.943)(3)\cos(4t-68.9°) = \underline{2.83\cos(4t-68.9°)}$

PROBLEM B.31 (Continued)

(d) $x(t) = 3e^{j4t} \Rightarrow s = j4 \therefore$ solution is the same as given for (c).

(e) $x(t) = 3\sin 4t = 3\cos(4t - 90°)$
$\Rightarrow s = j4$

(i) $H(j4) = 1.768 \angle -45°$ ~ from (c)(i)
$\Rightarrow y(t) = 5.303\cos(4t - 90 - 45°)$
$\underline{y(t) = 5.303\sin(4t - 45°)}$

(ii) $H(j4) = 0.943 \angle -68.9°$ ~ from (c)
$\Rightarrow y(t) = 2.83\cos(4t - 90° - 68.9°)$
$\underline{y(t) = 2.83\sin(4t - 68.9°)}$

(f) $x(t)$ of (e) is that of (c) delayed by $90°$.
$\therefore y(t)$ of (e) is that of (c) delayed by $90°$.

(g) (i) Char. eqn: $s + 4 = 0 \Rightarrow s = -4$
mode: $e^{-4t} = e^{-t/\tau} \Rightarrow \tau = \frac{1}{4}(s)$.

(ii) Char. eqn: $s^2 + 2s + 10 \Rightarrow s_{1,2} = -1 \pm j\sqrt{24}$
<u>Modes</u>: $e^{-t}e^{j\sqrt{24}}, e^{-t}e^{-j\sqrt{24}}$
$e^{-t} = e^{-t/\tau} \Rightarrow \tau = 1(s)$

(ii) for the system of (a)(i) $T_s \simeq 4\tau \simeq 1(s)$
for the system of (a)(ii) $T_s \simeq 4\tau = 4(s)$

PROBLEM 3.32

$x(t) = 2\cos 4t \Rightarrow s = j4$

(a) $H(j4) = \dfrac{K}{j4+a}$, $H(j4)x(t) = 5\cos(4t-45^\circ)$

$$\Rightarrow H(j4) = \frac{5}{2}\angle -45^\circ = \frac{K}{\sqrt{4^2+a^2}}\angle -\tan^{-1}\left(\frac{4}{a}\right)$$

$$\therefore \frac{K}{\sqrt{4^2+a^2}} = \frac{5}{2} \text{ and } \underline{a=4}.$$

$$K = \left(\sqrt{4^2+4^2}\right)\left(\frac{5}{2}\right) = \underline{14.142}$$

(b)
```
>> n = [0 14.142]; d = [1 4];
>> h = polyval(n, 4*j)/polyva (d, 4*j);
>> ymag = 2*abs(h);
>> y phase = angle (h)*180/pi
```

(c) $x(t) = 2\cos 3t \Longrightarrow s = j3$

$H(j3) = \dfrac{K}{j3+a}$; $H(j3)x(t) = 2.222\cos(3t-56.31^\circ)$

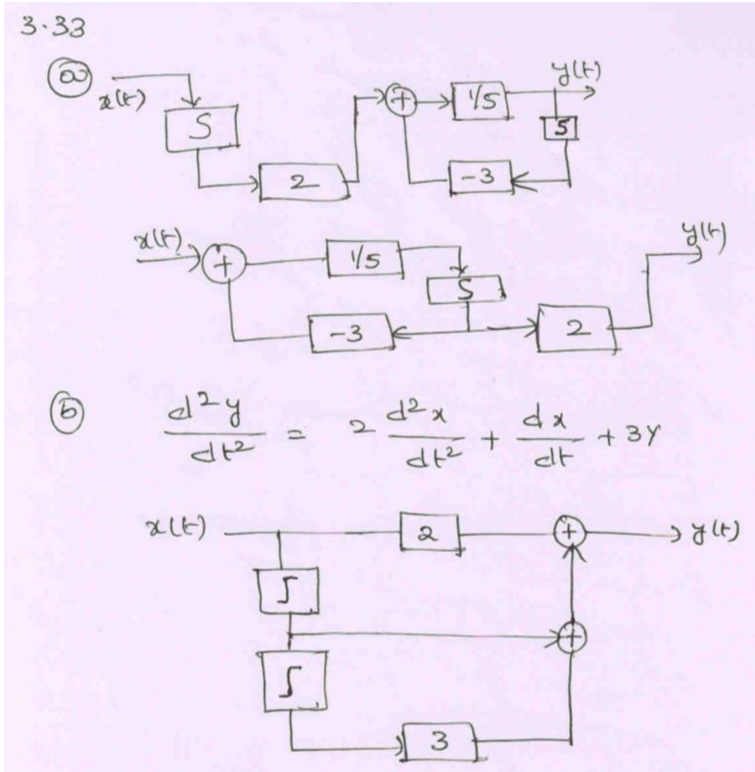
$$\Rightarrow H(j4) = \frac{2.222}{2}\angle -56.31^\circ = \frac{K}{\sqrt{3^2+a^2}}\angle -\tan^{-1}\left(\frac{3}{a}\right)$$

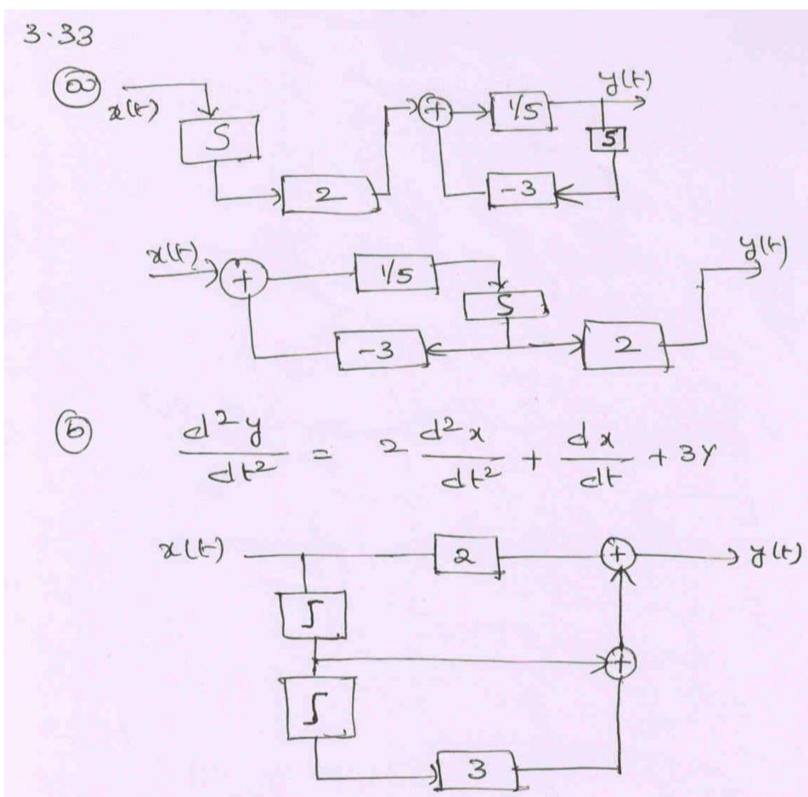
$$\therefore \tan^{-1}\left(\frac{3}{a}\right) = 56.31^\circ \Rightarrow a = \frac{3}{\tan(56.31^\circ)} = 2$$

$$\frac{K}{\sqrt{3^2+2^2}} = 1.111 \Rightarrow K = 1.111\sqrt{13} \simeq 4$$

$$H(s) = \frac{4}{s+2}$$

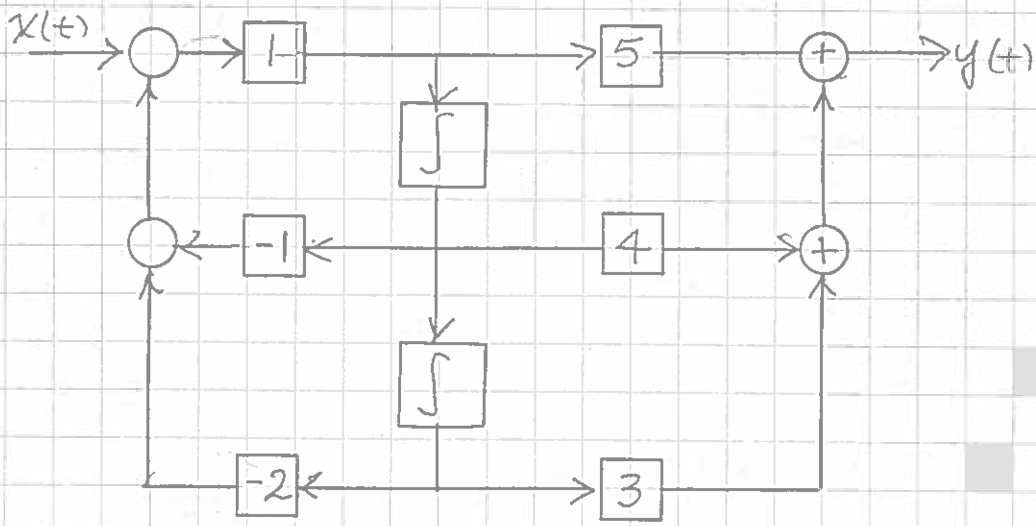
3.33.


3.33
(a)
x(t)
S
2
+
1/s
y(t)
S
-3
x(t)
+
1/s
S
-3
2
y(t)
(b)
$\frac{d^2y}{dt^2} = 2\frac{d^2x}{dt^2} + \frac{dx}{dt} + 3y$
x(t)
2
+
y(t)
∫
+
∫
3


3.33
(a)
x(t)
S
2
+
1/s
y(t)
S
-3
x(t)
+
1/s
S
-3
2
y(t)
(b)
$\frac{d^2y}{dt^2} = 2\frac{d^2x}{dt^2} + \frac{dx}{dt} + 3y$
x(t)
2
+
y(t)
∫
+
∫
3

PROBLEM 3.34

(a) Redraw the system diagram as:



(b) It can be seen that this is a Direct Form II System diagram.

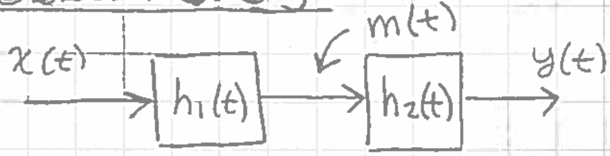
(a) From the diagram: $1/a_2 = 1$, $a_1 = 1$, $a_0 = 2$

$b_2 = 5$, $b_1 = 4$, $b_0 = 3$

$$\therefore \frac{d^2y(t)}{dt^2} + \frac{dy(t)}{dt} + 2y(t) = 5\frac{d^2x(t)}{dt^2} + 4\frac{dx(t)}{dt} + 3x(t)$$

PROBLEM 3.35

(a)

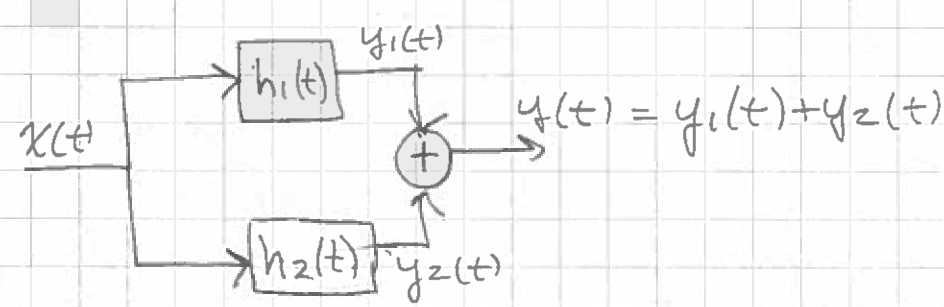


$$m(t) = x(t)H_1(s)$$

$$y(t) = H_2(s)m(t) = H_2(s)H_1(s)x(t) = H(s)x(t)$$

$$\therefore \; H(s) = H_1(s)H_2(s)$$

(b)



$$y_1(t) = H_1(s)x(t) \quad , \quad y_2(t) = H_2(s)x(t)$$

$$y(t) = \left[H_1(s) + H_2(s)\right]x(t) = H(s)x(t)$$

$$H(s) = H_1(s) + H_2(s)$$

PROBLEM 3.36

(a) From solution to Problem 3.12(a):

$$h(t) = h_1(t) * h_2(t) + h_1(t) * h_3(t) * h_4(t) + h_4(t) * h_5(t)$$

$$y(t) = h(t) * x(t) \; ; \; y_{ss}(t) = H(s)\, x(t)$$

$$H(s) = H_1(s)H_2(s) + H_1(s)H_3(s)H_4(s) + H_4(s)H_5(s)$$

(b) From the solution to Problem 3.13(a):

$$h(t) = h_1(t) * h_2(t) + h_1(t) * h_3(t) * h_4(t) + h_1(t) * h_3(t) * h_5(t)$$

$$y(t) = h(t) * x(t) \; ; \; y_{ss}(t) = H(s)\, x(t)$$

$$H(s) = H_1(s)H_2(s) + H_1(s)H_3(s)H_4(s) + H_1(s)H_3(s)H_5(s)$$

(c) From the solution to Problem 3.26:

$$y(t) = h_1(t) * x(t) - h_1(t) * h_2(t)\, y(t)$$

let, $m(t) = h_2(t) * y(t)$

then $y(t) = h_1(t) * [x(t) + m(t)]$

$$y_{ss}(t) = H_1(s)\,[x(t) + m(t)]$$

$$m_{ss}(t) = H_2(s)\, y_{ss}(t)$$

$$\Rightarrow y_{ss}(t) = H_1(s)x(t) + H_1(s)H_2(s)\, y_{ss}(t)$$

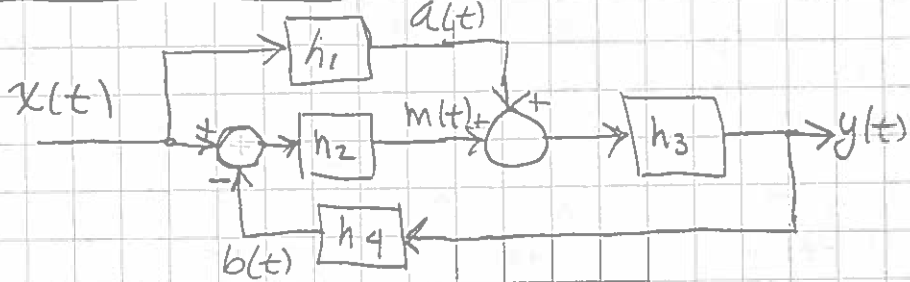
$$[1 + H_1(s)H_2(s)]\, y_{ss}(t) = H_1(s)\, x(t)$$

$$y_{ss}(t) = \frac{H_1(s)}{1 + H_1(s)H_2(s)}\, x(t) = H(s)\, x(t)$$

$$H(s) = \frac{H_1(s)}{1 + H_1(s)H_2(s)}$$

PROBLEM 3.37

(a)



$$a(t) = h_1(t) * x(t)$$

$$b(t) = h_4(t) * y(t)$$

$$m(t) = h_2(t) * [x(t) - b(t)] = h_2(t) * x(t) - h_2(t) * b(t)$$

$$= h_2(t) * x(t) - h_2(t) * h_4(t) * y(t)$$

$$y(t) = h_3(t) * [a(t) + m(t)]$$

$$= h_3(t) * h_1(t) * x(t) + h_3(t) * h_2(t) x(t)$$

$$- h_3(t) * h_2(t) * h_4(t) y(t)$$

$$= h_3(t) * [h_1(t) + h_2(t)] x(t) - h_3(t) * h_2(t) * h_4(t) y(t)$$

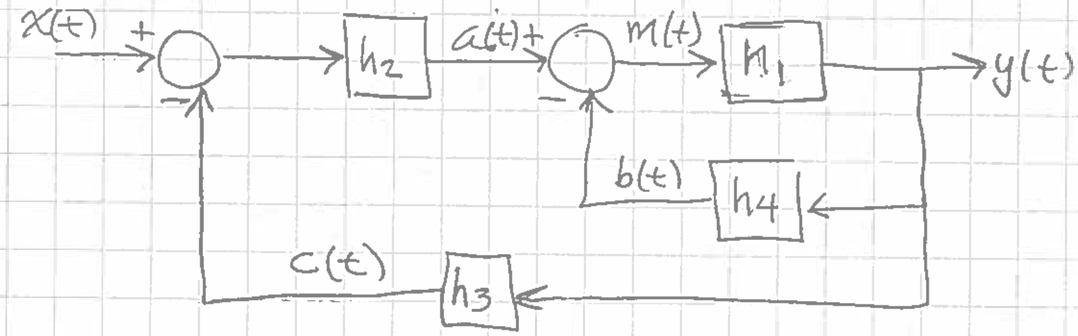
$$y_{ss}(t) = H_3(s)[H_1(s) + H_2(s)] x(t) - H_3(s) H_2(s) H_4(s) y_{ss}(t)$$

$$y_{ss}(t) = \frac{H_3(s)[H_1(s) + H_2(s)]}{1 + H_3(s) H_2(s) H_4(s)} x(t) = H(s) x(t)$$

$$H(s) = \frac{H_3(s)[H_1(s) + H_2(s)]}{1 + H_3(s) H_2(s) H_4(s)}$$

PROBLEM 3.37 (continued)

(b)



$$a(t) = h_2(t) * [x(t) - c(t)]$$

$$m(t) = a(t) + b(t)$$

$$b(t) = h_4(t) * y(t)$$

$$c(t) = h_3(t) * y(t)$$

$$\therefore \quad a(t) = h_2(t) * x(t) - h_2(t) * h_3(t) * y(t)$$

$$m(t) = h_2(t) * x(t) - h_2(t) * h_3(t) * y(t) - h_4(t) * y(t)$$

$$= h_2(t) * x(t) - [h_2(t) * h_3(t) + h_4(t)] * y(t)$$

$$y(t) = h_1(t) * m(t)$$

$$= h_1(t) * h_2(t) * x(t) - h_1(t) * [h_2(t) * h_3(t) + h_4(t)] * y(t)$$

$$y_{ss}(t) = H_1(s) H_2(s) x(t) - H_1(s) [H_2(s) H_3(s) + H_4(s)] y_{ss}(t)$$

$$y_{ss}(t) + H_1(s) [H_2(s) H_3(s) + H_4(s)] y_{ss}(t) = H_1(s) H_2(s) x(t)$$

$$y_{ss}(t) = \frac{H_1(s) H_2(s)}{1 + H_1(s) [H_2(s) H_3(s) + H_4(s)]} x(t)$$

$$= H(s) x(t)$$

$$\therefore \quad H(s) = \frac{H_1(s) H_2(s)}{1 + H_1(s) [H_2(s) H_3(s) + H_4(s)]}$$