## PROBLEM 10.1

(a) $x_a[n] = 2\delta[n+1] + 2\delta[n] - 4\delta[n-1] - 4\delta[n-2]$

(b) $x_b[n] = -2\delta[n+1] + 2\delta[n] + 2\delta[n-1]$

(c) $x_c[n] = -2\delta[n+1] + 4\delta[n] - 4\delta[n-1] + 4\delta[n-3]$

(d) $x_d[n] = 2\delta[n+1] + 2\delta[n-1] + 4\delta[n-3]$

## PROBLEM 10.2

(a) $x_a[n] = \delta[n] - \delta[n-1] + \delta[n-2] - \delta[n-3]$

(b) $x_b[n] = 2\delta[n] + 1.5\delta[n-1] + \delta[n-2] + 0.5\delta[n-3]$

(c) $x_c[n] = \delta[n] + \delta[n-1] - \delta[n-2] - \delta[n-3]$

(d) $x_d[n] = -\delta[n] - \delta[n-1] + \delta[n-2] + \delta[n-3]$

## PROBLEM 10.3

(a) $x[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2] + 4\delta[n-3] + 5\delta[n-4]$

(b) $x[n] \doteq 5\delta[n] + 2.5\delta[n-1] + 1.75\delta[n-2] + 1.25\delta[n-3]$
$\quad + \delta[n-4] + 0.9\delta[n-5] + 0.8\delta[n-6] + 0.6\delta[n-7] + \cdots$

(c) $x[n] = 1.5\delta[n+2] + \delta[n+1] + 0.5\delta[n] - 0.5\delta[n-2]$
$\quad - \delta[n-3] - 1.5\delta[n-4]$

## PROBLEM 10.4

$$g[n] * \delta[n] = \sum_{k=-\infty}^{\infty} g[k]\delta[n-k] \; ; \quad \delta[n-k] = \begin{cases} 1, & k=n \\ 0, & \text{otherwise} \end{cases}$$

$$\therefore \; g[n] * \delta[n] = g[n]$$

## PROBLEM 10.5

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

let $k = n - k'$, then $n - k = n - n + k' = k'$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k] \quad \checkmark$$

## PROBLEM 10.6

(a) $$y[n] = \sum_{k=-\infty}^{\infty} \alpha^k u[k] \beta^{n-k} u[n-k] = \left[\sum_{k=0}^{n} \alpha^k \beta^{n-k}\right] u[n]$$

$$= \beta^n \left[\sum_{k=0}^{n} \left(\frac{\alpha}{\beta}\right)^k\right] u[n] = \beta^n \left[\frac{1-\alpha^{n+1}\beta^{-(n+1)}}{1-\alpha\beta^{-1}}\right] u[n]$$

$$y[n] = \frac{\beta^{n+1} - \alpha^{n+1}}{\beta - \alpha} u[n]$$

(b) $$y[4] = \frac{\beta^5 - \alpha^5}{\beta - \alpha} = \beta^4 + \alpha\beta^3 + \alpha^2\beta^2 + \alpha^3\beta + \alpha^4$$

$$\begin{array}{r|l} & \beta^4 + \alpha\beta^3 + \alpha^2\beta^2 + \alpha^3\beta + \alpha^4 \\ \beta - \alpha & \beta^5 - \alpha^5 \\ & \beta^5 - \alpha\beta^4 \\ & \alpha\beta^4 - \alpha^5 \\ & \alpha\beta^4 - \beta^3\alpha^2 \\ & \beta^2\alpha^2 - \alpha^5 \\ & \cdots \end{array}$$

(c) $$y[4] = \sum_{k=0}^{4} \alpha^k \beta^{4-k} = \alpha^0\beta^4 + \alpha^1\beta^3 + \alpha^2\beta^2 + \alpha^3\beta^1 + \alpha^4\beta^0$$

$$y[4] = \beta^4 + \alpha\beta^3 + \alpha^2\beta^2 + \alpha^3\beta + \alpha^4 \quad \checkmark$$

## PROBLEM 10.7

(a) $y[5] = \sum_{k=-\infty}^{\infty} x[k]h[5-k] = \sum_{k=1}^{6} h[5-k] = h[4] + h[3] + h[2] + h[1] + h[0] + h[-1] = 3 \cdot 2 = 6$

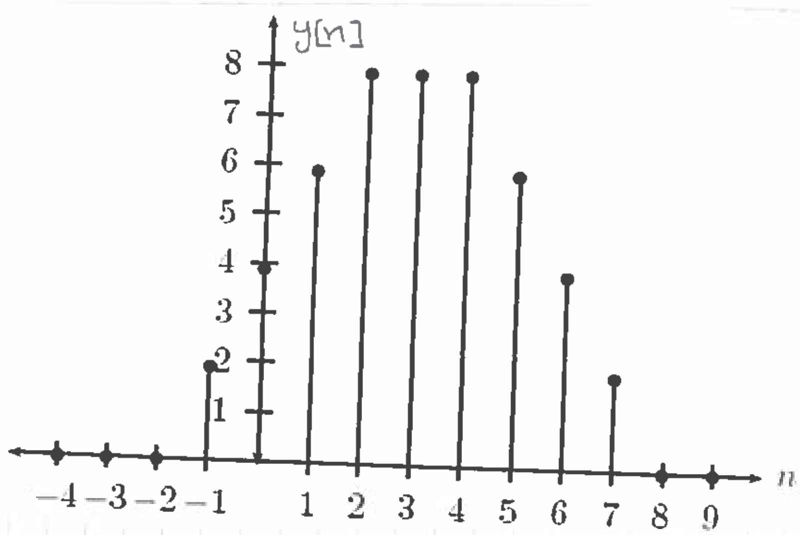
(b) max is $h[1] + h[0] + h[-1] + h[-2] = 8$

(c) max occurs at $n = 2, 3, 4$

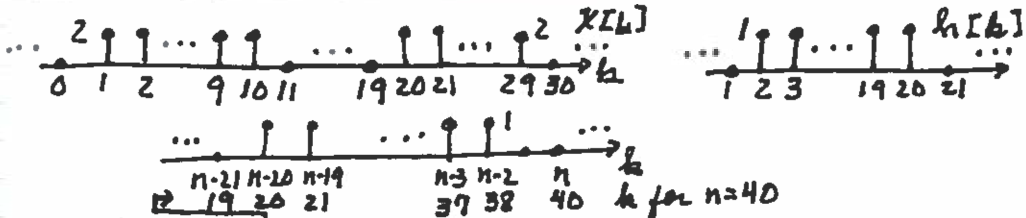
(d)

$$
\begin{aligned}
y[n] &= \sum_{k=-\infty}^{\infty} x[k]h[n-k] \\
&= 0, n \leq -2 \\
&= h[-2] = 2, n = -1 \\
&= h[-2] + h[-1] = 4, n = 0 \\
&= h[-2] + h[-1] + h[0] = 6, n = 1 \\
&= h[-2] + h[-1] + h[0] + h[1] = 8, n = 2, 3, 4 \\
&= h[-1] + h[0] + h[1] = 6, n = 5 \\
&= h[0] + h[1] = 4, n = 6 \\
&= h[1] = 2, n = 7 \\
&= 0, n \geq 8
\end{aligned}
$$

(e)
```
>>n=-2:8
>>x=[0,0,0,1,1,1,1,1,1,0,0];
>>h=[2,2,2,2,0,0,0,0,0,0,0];
>>y=conv(x,h);
>>stem((-2+-2):(8+8),y); title('y[n]'), xlabel('n')
```



## PROBLEM 10.8

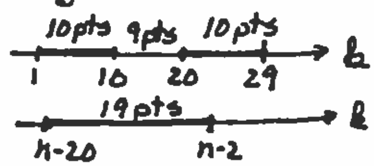



(a) At $n=40$:

overlap from $k=20$ to $k=29$, or 10 points

$\therefore y[40] = 2(10) = \underline{20}$

(b)




$\therefore$ Maximum points overlap $= 10$

$\therefore y_{max} = 2(10) = \underline{20}$

(c) Begins for $n-2=10$, or $n=12$ $\quad \therefore 12 \le n \le 40$

Ends after $n-20=20$, or $n=40$

(d) $y[n]$ increases linearly from $n-2=1$ $[n=3]$ to $n-2=10$ $[n=12]$

$\therefore \; y[n] = 0, \quad n < 3$

$y[n] = 2(n-2), \quad 3 \le n \le 12$

$y[n] = 2(20), \quad 12 \le n \le [(n-20) = 20] \Rightarrow 12 \le n \le 40$

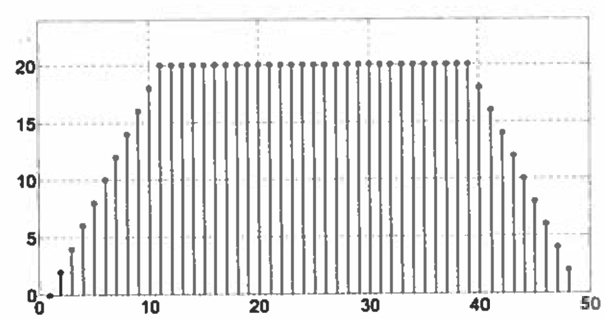
$y[n] = 2(50-n), \quad 41 \le n \le 50$

$y[n] = 0, \quad 50 \le 50$

$y[n]$ begins decreasing linearly at $n-20=21$, or $\underline{n=41}$, and equals zero at $n-20=30$, or $\underline{n=50}$.

(e)

```
>> for n=1:10
x(n)=2;
x(n+19)=2;
end
>> for n=2:20
h(n)=1;
end
>> y=conv(x,h);
>> stem(y)
```



# PROBLEM 10.9

(a) $y[n] = \sum_{k=-\infty}^{\infty} (u[k] - u[k-2])(x[n-k])$

$= \sum_{k=0}^{1} x[n-k] = \sum_{k=0}^{1} \delta[n+2-k] + 3(0.7)^{(n-k)}(u[n-k] - u[n-5-k])$

$= \delta[n+2] + \delta[n+1] + 3(0.7)^n(u[n] - u[n-5])$
$\quad + 3(0.7)^{n-1}(u[n-1] - u[n-6]$

(b) $y[-1] = 0 + 1 + 0 + 0 = \underline{1}$

$y[0] = 0 + 0 + 3(0.7)^0(1-0) + 0 = \underline{3}$

$y[3] = 3(0.7)^3(1-0) + 3(0.7)^2(1-0)$
$\quad = 3(0.7)^2[0.7+1] = \underline{2.499}$

$y[10] = 0 + 0 + 0 + 0 = \underline{0}$

# PROBLEM 10.10

(a) $y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k] = \sum_{k=0}^{1} x[n-k] + \sum_{k=4}^{5} x[n-k]$

$x[n-k] = u[n-k]$

$\therefore y[n] = u[n] + u[n-1] + u[n-4] + u[n-5]$

$\therefore y[n] = 0, \ n<0$

| | |
|---|---|
| $y[0] = 1$ | $y[3] = 2$ |
| $y[1] = 2$ | $y[4] = 3$ |
| $y[2] = 2$ | $y[n] = 4, \ n \geq 5$ |

(b) $y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k] = \sum_{k=0}^{1} (u[n-k] - u[n-2-k]) + \sum_{k=4}^{5} (\quad)$

$= u[n] - u[n-2] + u[n-1] - u[n-3]$
$\quad + u[n-4] - u[n-6] + u[n-5] - u[n-7]$

$y[n] = 0, \ n<5$

| | |
|---|---|
| $y[0] = 1$ | $y[4] = 1$ |
| $y[1] = 2$ | $y[5] = 2$ |
| $y[2] = 1$ | $y[6] = 1$ |
| $y[3] = 0$ | $y[n] = 0 \quad n \geq 7$ |

(c)
```
x=[1 1 0 0 0 0];h=[1 1 0 0 1 1];
y=conv(x,h)
```

(d) $y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k] = \sum_{k=0}^{1} (u[n-k] - u[n-6-k]) + \sum_{k=4}^{5} (\quad)$

$= u[n] - u[n-6] + u[n-1] - u[n-7]$
$\quad + u[n-4] - u[n-10] + u[n-5] - u[n-11]$

$y[n] = 0, \ n<0$

| | | |
|---|---|---|
| $y[0] = 1$ | $y[4] = 3$ | $y[8] = 2$ |
| $y[1] = 2$ | $y[5] = 4$ | $y[9] = 2$ |
| $y[2] = 2$ | $y[6] = 3$ | $y[10] = 1$ |
| $y[3] = 2$ | $y[7] = 2$ | $y[n] = 0, \ n \geq 11$ |

## PROBLEM 10.10 (continued)

**(e)**

```
x=[1 1 1 1 1 1 0 0];h=[1 1 0 0 1 1]; y=conv(x,h)
```

**(f)** $y[n]=\sum_{k=0}^{1}(u[n-k]-u[n-k-2])=u[n]+u[n-1]-u[n-2]-u[n-3]$

$\therefore\ y[n]=0,\ n<0$

$y[0]=1$

$y[1]=2$

$y[2]=1$

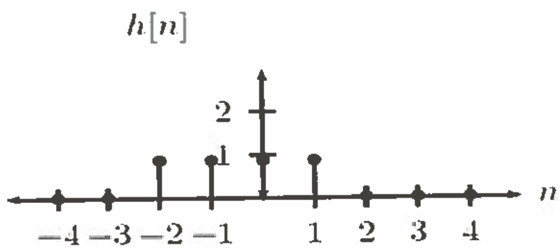
$y[n]=0\ ,\ n\geq 3$

**(g)**

```
x=[1 1 0 0];h=[1 1 0 0]; y=conv(x,h)
```

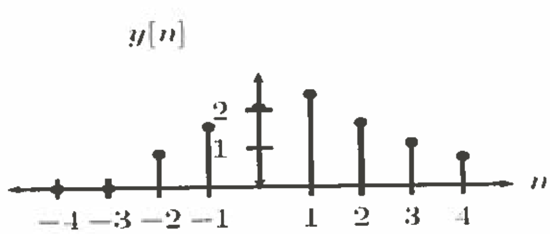
## PROBLEM 10.11

**(a)**




**(b)** $y[n]=\sum_{k=-\infty}^{\infty}h[k]x[n-k]$ and since $h[k]=0$ outside of $k\in[-2,1]$, we have:

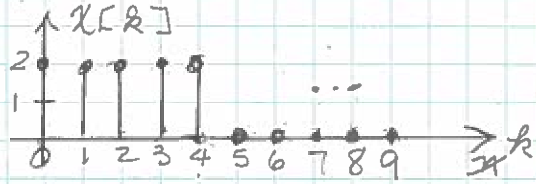
$$
\begin{aligned}
y[n] &= \sum_{k=-2}^{1}1x[n-k]=\sum_{k=-2}^{1}(0.7)^{n-k}u[n-k] \\
&= 0,\ n\leq -3 \\
&= (0.7)^0=1,\ n=-2 \\
&= (0.7)^1+(0.7)^0=1.7,\ n=-1 \\
&= (0.7)^2+(0.7)^1+1=2.19,\ n=0 \\
&= (0.7)^{n+2}+(0.7)^{n+1}+(0.7)^n+(0.7)^{n-1},\ n\geq 1
\end{aligned}
$$

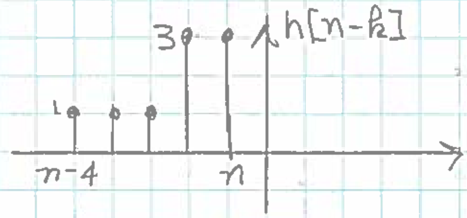


# PROBLEM 10.12

(a) $x[n] = [2\ 2\ 2\ 2\ 2\ \ 0\ 0\ 0]$

$h[n] = [3\ 3\ 1\ 1\ 1\ \ 0\ 0\ 0]$




$y[n] = 0, \quad n < 0$

$y[0] = (2)(3) = 6$

$y[1] = (2)(3) + (2)(3) = 12$

$y[2] = (2)(3) + (2)(3) + (2)(1) = 14$

$y[3] = (2)(3) + (2)(3) + (2)(1) + (2)(1) = 16$

$y[4] = (2)(3) + (2)(3) + (2)(1) + (2)(1) + (2)(1) = 18.$

$y[5] = (0)(3) + (2)(3) + (2)(1) + (2)(1) + (2)(1) + (2)(0) = 12$

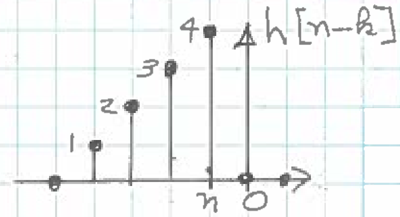
$y[6] = (0)(3) + (0)(3) + (2)(1) + (2)(1) + (2)(1) + (2)(0) = 6$

$y[7] = (0)(3) + (0)(3) + (0)(1) + (2)(1) + (2)(1) + (2)(0) + (2)(0) = 4$

$y[8] = (0)(3) + (0)(3) + (0)(1) + (0)(1) + (2)(1) + (2)(0) = 2$

$y[n] = 0, \quad n \geq 9$

(b) $y[n] = 0, \quad n < 0$




$y[0] = (4)(2) = 8$

$y[1] = (4)(2) + (3)(2) = 14$

$y[2] = (4)(2) + (3)(2) + (2)(2) = 18$

$y[3] = (4)(2) + (3)(2) + (2)(2) + (1)(2) = 20$

$y[4] = (4)(2) + (3)(2) + (2)(2) + (1)(2) + (0)(2) = 20$

$y[5] = (4)(0) + (3)(2) + (2)(2) + (1)(2) + (0)(2) = 12$

$y[6] = (4)(0) + (3)(0) + (2)(2) + (1)(2) + (0)(2) = 6$

$y[7] = (4)(0) + (3)(0) + (2)(0) + (1)(2) + (0)(2) = 2$

$y[n] = 0, \quad n \geq 8.$

(c) $y[n] = [-4\ \ -8\ \ -4\ \ 0\ \ 0\ \ 4\ \ 8\ \ 4\ \ 0\ \ 0\ \cdots]$

(d) $y[n] = [6\ \ 0\ \ 2\ \ 0\ \ -4\ \ -2\ \ 0\ \ -2\ \ 0\ 0\ 0 \cdots]$

(e) $y[n] = [8\ \ 14\ \ 2\ \ -8\ \ -8\ \ -6\ \ -2\ \ 0\ 0\ 0 \cdots]$

Solutions (c)-(f) follow the same process as (a) & (b) shown above

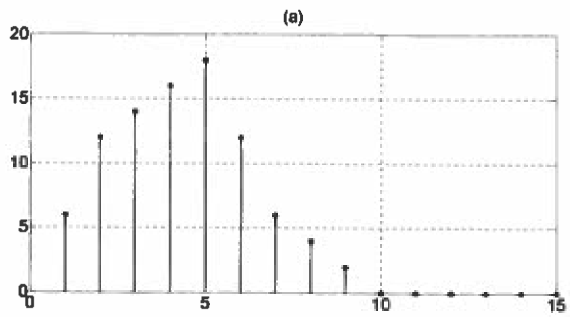
## PROBLEM 10.12 (continued)

(g)

```
>> a=[2 2 2 2 2 0 0 0];  b=[3 3 1 1 1 0 0 0];   c=[2 -2 2 -2 0 0 0 0];
>> d=[4 3 2 1 0 0 0 0];  e=[2 2 -2 -2 0 0 0 0]; f=[-2 -2 2 2 0 0 0 0];
```

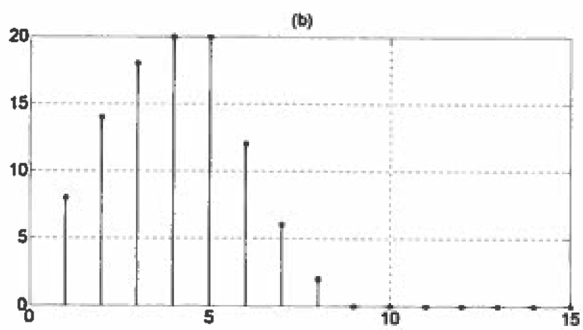
(a)

```
>> stem(a)
>> x=a;  h=b
>> y=conv(x,h);
>> stem(y)
```



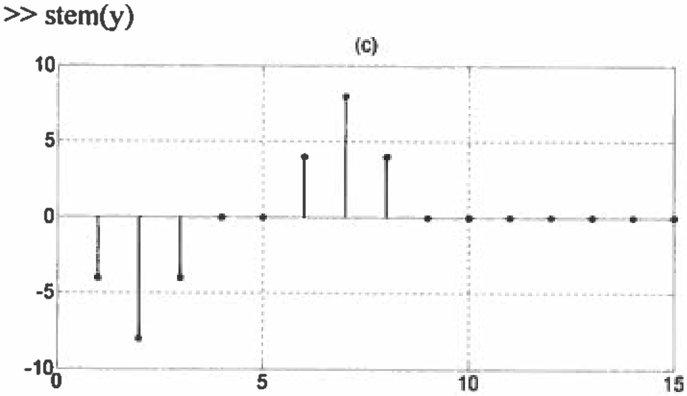

(b)

```
>> x=a; h=d;
>> y=conv(x,h);
>> stem(y)
```




(c)

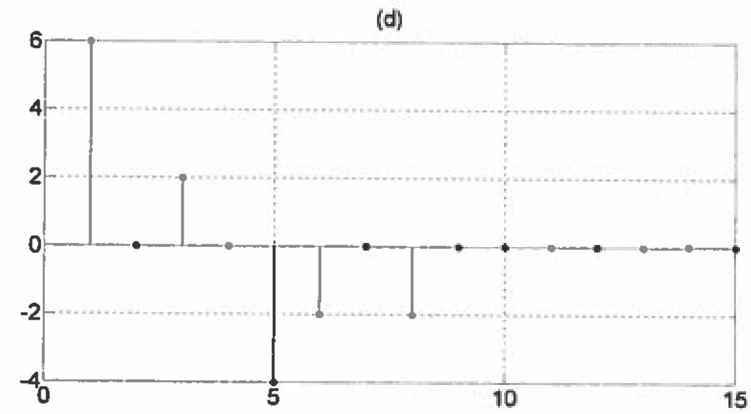
```
>> x= a;  h=f;
>> y=conv(x,h);
>> stem(y)
```



**PROBLEM 10.12** (continued)

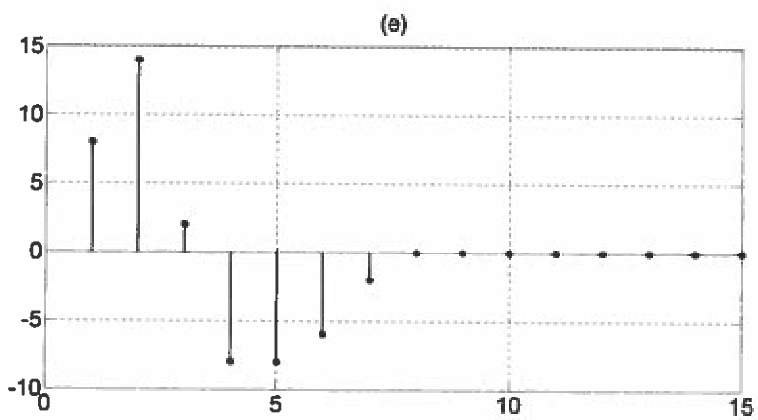
(d)

```
>> x=c; h=b;
>> y=conv(x,h);
>> stem(y)
```



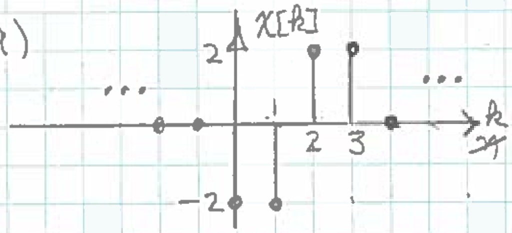

(e)

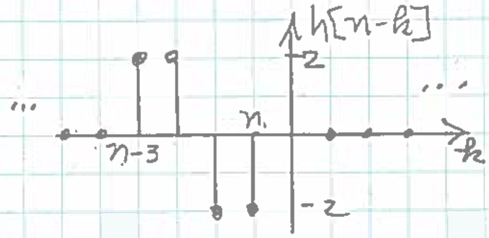
```
>>  x=e; h=d;
>> y=conv(x,h);
>> stem(y)
```



## PROBLEM 10.13

(a)




(b) $y[n] = 0, \; n < 0$

$y[0] = (-2)(-2) = 4$

$y[1] = (-2)(-2) + (-2)(-2) = 8$

$y[2] = (2)(2) + (-2)(-2) + (2)(-2) = -4$

$y[3] = (-2)(2) + (-2)(2) + (2)(-2) + (2)(-2) = -16$

$y[4] = (-2)(0) + (-2)(2) + (2)(2) + (2)(-2) = -4$

$y[5] = (-2)(2) + (2)(2) = 8$

$y[6] = (2)(2) = 4$

$y[n] = 0, \; n \geq 7$

## PROBLEM 10.14

(a) $h[n] = h_1[n] * h_2[n] = \sum_{k=-\infty}^{\infty} (0.9)^k u[k] (0.9)^{(n-k)} u[n-k]$

$= \sum_{k=0}^{n} (0.9)^k (0.9)^{n-k} = (0.9)^n \sum_{k=0}^{n} (0.9)^{-(k-n)} (0.9)^{(n-k)}$

$= 0.9^n \left( \sum_{k=0}^{n} 1 \right) u[n] = n (0.9)^n u[n]$

(b) $h[n] = \delta[n-2] * \delta[n-2] = \delta[n-4]$

(c) $h[n] = \sum_{k=-\infty}^{\infty} (u[k-1] - u[k-3]) (u[n-1-k] - u[n-3-k])$

$= \sum_{k=1}^{2} (u[n-1-k] - u[n-3-k])$

$= u[n-1-1] - u[n-3-1] + u[n-1-2] - u[n-3-2]$

$= u[n-2] - u[n-4] + u[n-3] - u[n-5]$

$= \delta[n-2] + 2\delta[n-3] + \delta[n-4]$

PROBLEM 10.15

$h[n] = (1.2)^n u[n]$

(a) $h[n] = 0,\ n < 0\ \therefore$ <u>causal</u>

(b) $\sum_{n=-\infty}^{\infty} |h[n]| = 1 + 1.2 + (1.2)^2 + (1.2)^3 + \cdots$ unbounded, <u>not stable</u>

(c) $y[n] = h[n] * u[n] = \sum_{k=-\infty}^{\infty} (1.2)^k u[k]\, u[n-k]$

$$= \sum_{k=0}^{n} (1.2)^k = \frac{1-(1.2)^{n+1}}{1-1.2} = \underline{5(1.2)^{n+1} - 5}$$

(d) $y[2] = 5(1.2)^n - 5 = \underline{3.64}$

```
x=[1 1 1 1 1 1];h=[1 1.2 1.2^2 1.2^3 1.2^4];
y=conv(x,h)
```

(e) $h[n] = (1.2)^n u[-n]$

(a) $h[n] \neq 0,\ n < 0$, <u>noncausal</u>

(b) $\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{k=-\infty}^{0} (1.2)^n = 1 + (1.2)^{-1} + [(1.2)^{-1}]^2 + \cdots$

$$= \sum_{k=0}^{\infty} [(1.2)^{-1}]^k = \frac{1}{1-\frac{1}{1.2}} = 6 < \infty, \ \underline{\text{stable}}$$

(c) $y[n] = 1.2^n u[n] * u[n] = \sum_{k=-\infty}^{\infty} 1.2^k u[-k]\, u[n-k]$

$$= \sum_{k=-\infty}^{0} 1.2^k u[n-k]$$

$$= \begin{cases} \sum_{k=-\infty}^{0} 1.2^k = \underline{6}, & n \geq 0 \\ \sum_{k=-\infty}^{-n} 1.2^k, & n < 0 \end{cases}$$

(f) $h[n] = (0.3)^n u[-n]$

(a) $h[n] \neq 0,\ n < 0\ \therefore$ not causal.

(b) $\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=-\infty}^{0} (0.3)^k = 1,\ 3.33,\ 11.11,\ 37.04, \ldots$ <u>unbounded</u> $\therefore$ <u>not stable</u>

(g) $h[n] = u[-n]$

(a) not causal

(b) $\sum_{k=-\infty}^{\infty} |h[n]| = \sum_{-\infty}^{0} (1)$ — unbounden $\therefore$ <u>not stable</u>

## PROBLEM 10.16

$$f[n]*g[n] = \sum_{m=-\infty}^{\infty} f[m]g[n-m] = e[n]$$

$$f[n]*g[n]*h[n] = \sum_{k=-\infty}^{\infty} e[k]h[n-k]$$

$$= \sum_{k=-\infty}^{\infty}\left[\sum_{m=-\infty}^{\infty} f[m]g[k-m]\right]h[n-k]$$

$$= \sum_{m=-\infty}^{\infty}\left[\sum_{k=-\infty}^{\infty} g[k-m]h[n-k]\right]f[m] \qquad \text{Let } k-m=p, \text{ or } k=m+p$$

$$\therefore \Rightarrow \sum_{m=-\infty}^{\infty}\left[\sum_{p=\infty}^{\infty} g[p]h[n-m-p]\right]f[m] \qquad \text{Let } q=n-p, \text{ or } p=n-q$$

$$\Rightarrow \sum_{m=-\infty}^{\infty}\left[\sum_{q=\infty}^{-\infty} g[n-q]h[q-m]\right]f[m]$$

$$= \sum_{q=-\infty}^{\infty}\left[\sum_{m=-\infty}^{\infty} f[m]h[q-m]\right]g[n-q] = f[n]*h[n]*g[n]$$

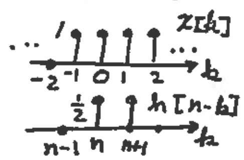
## PROBLEM 10.17

$$y[n] = \frac{1}{2}(x[n+1] + x[n])$$

(a) $h[n] = y[n]\Big|_{x[n]=\delta[n]} = \frac{1}{2}(\delta[n+1]+\delta[n]) = \begin{cases} 0.5, & n=-1 \\ 0.5, & n=0 \\ 0, & \text{otherwise} \end{cases}$

(b) noncausal, $h[n] \neq 0,\ n<0$

(c)




$y[n]=0,\ n<-2$

$y[-2]=\frac{1}{2}$

$y[n]=1,\ n\geq -1$

(d) $\delta[n-1]*[-h[n]] = -h[n-1]$

$\therefore h_t[n] = h[n] - h[n-1] = \frac{1}{2}\delta[n+1] + \frac{1}{2}\delta[n] - \frac{1}{2}\delta[n] - \frac{1}{2}\delta[n-1]$

$= \frac{1}{2}(\delta[n+1] - \delta[n-1]) = \begin{cases} 0.5, & n=-1 \\ 0, & n=0 \\ -0.5, & n=1 \\ 0, & \text{otherwise} \end{cases}$

(e) (i) From (c),

$y[n]=0,\ n<-2$

$y[-2]=\frac{1}{2}$

$y[-1] = 1 - \frac{1}{2} = \frac{1}{2}$

$y[n] = 1-1 = 0,\ n\geq 0$

(ii) For $n\geq -2$, $y[n] = h[-1]x[n+1] + h[1]x[n-1]$

$y[-2] = h[-1]x[-1] + h[1]x[-3] = \frac{1}{2}$

$y[-1] = h[-1]x[0] + h[1]x[-2] = 1/2$

$y[0] = h[-1]x[1] + h[1]x[-1] = 0$, $y[n]=0,\ n\geq 0$

## PROBLEM 10.18

(a) linear, $\cos(0.1\pi n)\,[x_1[n] + x_2[n]] = y_1[n] + y_2[n]$

(b) Time-varying, $y[n]\Big|_{n \leftarrow n-n_0} \neq y[n]\Big|_{x[n] \leftarrow x[n-n_0]}$

(c) $y[n]\Big|_{x[n]=\delta[n]} = h[n] = \cos(0.1\pi n)\,\delta[n] = 1\,\delta[n]$

(d) $y[n] = \cos(0.1\pi n)\,\delta[n-1] = \cos(0.1\pi)\,\delta[n-1] = 0.951\,\delta[n-1]$

(e) No. $h[n]$ does not describe $h[n-1]$, etc.

## PROBLEM 10.19

(a) Causal, $h[n]=0$ for $n<0$

$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=-\infty}^{\infty} e^{-3n} u[n-1] = \sum_{n=1}^{\infty} (e^{-3})^n$$

$$= \sum_{n=0}^{\infty} (e^{-3})^n - 1 \overset{\text{App C}}{=} \frac{1}{1-e^{-3}} - 1 < \infty \quad \underline{\text{stable}}$$

(b) noncausal, $h[n] \neq 0$, $n<0$

$$\sum_{n=-\infty}^{\infty} e^{3n} u[1-n] = \sum_{n=-\infty}^{1} e^{3n} = \sum_{k=-1}^{\infty} e^{-3k}$$

$$= e^{3} + \sum_{k=0}^{\infty} (e^{-3})^k = e^3 + \frac{1}{1-e^{-3}} < \infty, \ \underline{\text{stable}}$$

(c) causal, $h[n]=0$, $n<0$

$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=0}^{\infty} e^{3n} = 1 + e^3 + e^6 + e^9 + \cdots$$

which is unbounded, $\therefore$ not stable

(d) causal, $h[n]=0$, $n<0$

$\sum_{n=-\infty}^{\infty} |\cos 3n|$ is unbounded, since $|\cos 3n|$ does not approach zero as $n \to \infty$, $\therefore$ not stable

(e) causal, $h[n]=0$, $n<0$

$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{k=0}^{\infty} n(e^{-3})^n = \frac{e^{-3}}{(1-e^{-3})^2} < \infty \ ; \therefore \ \underline{\text{stable}}$$

(f) causal, $h[n]=0$, $n<0$

$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=0}^{\infty} |e^{-n} \cos 3n|$$

Since $|\cos 3n| < 1$, $|e^{-n} \cos 3n| \leq e^{-n}$

$$\therefore \sum_{n=-\infty}^{\infty} |h[n]| \leq \sum_{k=0}^{\infty} (e^{-1})^n = \frac{1}{1-e^{-1}} < \infty, \ \therefore \ \underline{\text{stable}}$$

## PROBLEM 10.20

(a) $y[n] = \sum_{n=-\infty}^{\infty} h[n-k]x[k] = \sum_{k=0}^{\infty} e^{-2(n-k)} x[k-1]$

let $x[n] = \delta[n]$, then

$$y[n] = h[n] = \sum_{k=0}^{\infty} e^{-2(n-k)}\delta[k-1] = e^{-2(n-1)}\delta[n-1]$$

(b) Causal, $h[n] = 0,\ n < 0$

(c) Stable, $\sum_{n=-\infty}^{\infty} |h[n]|$ is finite

(d) $y[n] = \sum_{k=-\infty}^{0} e^{-2k} x[n-k] = \sum_{k=-\infty}^{\infty} e^{-2k} u[-k]\, x[n-k]$

$\therefore h[n] = e^{-n} u[-n]$ — non causal

$$\sum_{n=-\infty}^{\infty} |e^{-2n} u[-n]| = \sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=-\infty}^{\infty} e^{-2n} = 1 + e^{2} + e^{4} + e^{6} + \cdots$$

unbounded

$\therefore$ not stable

## PROBLEM 10.21

(a) $h[n] = \delta[n+7] + \delta[n-7]$

(b) $h[n] = \sum_{k=-\infty}^{n-3} \sum_{k=n}^{\infty} \delta[k-2]$

for $n \le 2$, $\sum_{k=n}^{\infty} \delta[k-2] = 1$

for $n > 2$, $\sum_{k=n}^{\infty} \delta[k-2] = 0$

$$h[n] = \sum_{k=-\infty}^{n-3} 1$$

## PROBLEM 10.22

(a) $y[n]\big|_{x[n]=\delta[n]} = h[n] = \underline{\delta[n-2]}$

(b) $y[n]\big|_{x[n]=\delta[n]} = \sum_{k=-\infty}^{n} \delta[k-5]$ , where $\delta[k-5] = \begin{cases} 1, & k=5 \\ 0, & \text{otherwise} \end{cases}$

$\therefore h[n] = \begin{cases} 1, & n \geq 5 \\ 0 & n < 5 \end{cases} = \underline{u[n-5]}$

(c) From (b), $\sum_{m=-\infty}^{k} \delta[m-5] = u[k-5]$

$\therefore h[n] = \sum_{k=-\infty}^{n} u[k-5]$ ; $u[k-5] = \begin{cases} 1, & k \geq 5 \\ 0, & k<5 \end{cases}$

$\therefore h[n] = (n-4)u[n-5] = \underline{(n-4)u[n-5]}$

## PROBLEM 10.23

(i)(a) $y[n] - 5/6\, y[n-1] = 2^n u[n]$ , $y[-1] = 0$

$z - 5/6 = 0 \Rightarrow z = 5/6$ ; $\underline{y_c[n] = C\left(5/6\right)^n}$

$y_p[n] = P(2)^n$

$P(2)^n - \frac{5}{6}P(2)^{n-1} = 2^n$

for $n=1$ , $2P - 5/6\,P = 2 \Rightarrow P = \frac{12}{7}$

$y[n] = C\left(\frac{5}{6}\right)^n + \left(\frac{12}{7}\right)2^n$

$y[-1] = 0 = \frac{6}{5}C + \frac{6}{7} \Rightarrow C = \frac{5}{6}\left(-\frac{6}{7}\right) = -\frac{5}{7}$

$\therefore y[n] = -\frac{5}{7}\left(\frac{5}{6}\right)^n + \left(\frac{12}{7}\right)2^n$

(b) Check $y[-1] = -\frac{5}{7}\left(\frac{6}{5}\right) + \frac{12}{7}\left(\frac{1}{2}\right) = 0$ ✓

(c)
```
y(1) = 1
for k=0:3
y(k+2) = 5*y(k+1)/6 + 2^(k+1);
w(k+1) = -(5/7)*(5/6)^k +(12/7)*2^k;
end
```

# PROBLEM 10.23 (continued)

(ii) (a) $z-0.7=0$, $\therefore y_c[n] = \underline{C(0.7)^n}$

$y_p[n] = P \Rightarrow P-0.7P=1 \Rightarrow P=\frac{1}{0.3} = \underline{3.333}$

$y[n] = 3.333 + C(0.7)^n$

$y[-1] = -3 = 3.333 + C(0.7)^{-1} \Rightarrow \frac{C}{0.7} = -6.333,\ C = \underline{-4.433}$

$\therefore y[n] = \underline{3.333 - 4.433(0.7)^n,\ n \geq -1}$

$y[3] = 3.333 - 4.433(0.7)^3 = 1.812$ ✓ checks MATLAB

(b) $y[-1] = 3.333 - 4.433/0.7 = \underline{-3}$ ✓

$\underline{n \geq 0}$ $y[n]-0.7y[n-1] = 3.333 - 4.433(0.7)^n$

$-0.7[3.333 - 4.333(0.7)^{n-1}] = 3.333 - 4.433(0.7)^n$

$-2.333 - 4.333(0.7)^n = 1$ ✓

(iii) (a) From (ii), $y_c[n] = C(0.7)^n$

$y_p[n] = Pe^{-n}$, $\therefore Pe^{-n} - 0.7Pe^{-(n-1)} = Pe^{-n}[1-0.7e^{1}]$

$= Pe^{-n}[-0.903] = e^{-n} \Rightarrow P = \underline{-1.108}$

$\therefore y[n] = \underline{C(0.7)^n - 1.108e^{-n}}$

$y[-1] = 0 = \frac{C}{0.7} - 1.108e \Rightarrow \frac{C}{0.7} = 3.012 \Rightarrow \underline{C = 2.108}$

$\therefore y[n] = \underline{-1.108e^{-n} + 2.108(0.7)^n}$, $n \geq -1$

(b) $y[-1] = -1.108e + 2.107/0.7 = -3.012 + 3.01 = 0$ ✓

$y[n]-0.7y[n-1] = -1.108e^{-n} + 2.108(0.7)^n$

$-0.7[-1.108e^{-(n-1)} + 2.108(0.7)^{n-1}] = -1.108e^{-n} + 2.108(0.7)^n$

$+2.108e^{-n} - 2.108(0.7)^n = e^{-n}$ ✓

(iv) (a) $z^2 - 1.7z + 0.72 = (z-0.9)(z-0.8)$

$\therefore y_c[n] = \underline{C_1(0.8)^n + C_2(0.9)^n}$

$y_p[n] = P$, $\therefore P - 1.7P + 0.72P = 0.02P = 1 \Rightarrow P = \underline{50}$

$\therefore y[n] = 50 + C_1(0.8)^n + C_2(0.9)^n$

$y[-1] = 0 = 50 + \frac{C_1}{0.8} + \frac{C_2}{0.9} \Rightarrow 1.25C_1 + 1.111C_2 = -50$

$y[-2] = 1 = 50 + \frac{C_1}{(0.8)^2} + \frac{C_2}{(0.9)^2} \Rightarrow 1.563C_1 + 1.235C_2 = -49$

$\therefore C_1 = \frac{\begin{vmatrix} -50 & 1.111 \\ -49 & 1.235 \end{vmatrix}}{\begin{vmatrix} 1.25 & 1.111 \\ 1.563 & 1.235 \end{vmatrix}} = \underline{37.92}$ ; $C_2 = \frac{\begin{vmatrix} 1.25 & 1.111 \\ 1.563 & 1.235 \end{vmatrix}}{\bullet} = \underline{-87.56}$

$\therefore y[n] = \underline{37.92(0.8)^n - 87.56(0.9)^n + 50}$

(b) $y[-1] = \frac{37.92}{0.8} - \frac{87.56}{0.9} + 50 = 0$ ✓

$y[-2] = 50 + \frac{37.92}{(0.8)^2} - \frac{87.56}{(0.9)^2} = 1.15$ ✓

$y[n] - 1.7y[n-1] + 0.72y[n-2] = 50 + 37.92(0.8)^n$

$-87.56(0.9)^n - 1.7(50) - 1.7(37.92)(0.8)^{n-1} + 1.7\left(\frac{87.56}{0.9}\right)(0.9)^n$

$+0.72(50) + 0.72\left(\frac{37.92}{0.64}\right)(0.8)^n - 0.72\left(\frac{87.56}{0.81}\right)(0.9)^n = 1$ ✓

PROBLEM 10.23 (continued)

(v)(a) From (ii), $y_c = C(0.7)^n$

$y_p[n] = P_1 \cos n + P_2 \sin n$

$\therefore\ y[n] - 0.7y[n-1] = P_1 \cos n + P_2 \sin n - 0.7P_1 \cos(n-1)$

$-0.7 P_2 \sin(n-1)$, $\sin(n-1)\big|_{n=0} = \sin(-57.3°)$

$\therefore\ 0.622P_1 + 0.581P_1 = 1$

$0.589P_1 + 0.622P_2 = 0$

$\therefore P_1 = \underline{12.61}$, $P_2 = \underline{-11.78}$

(c)

```
y(1)=-3;
for n=1:5
  y(n+1)=.7*y(n)+1
end
```

PROBLEM 10.24

(a) $2y[n] - y[n-1] + 4y[n-2] = 5x[n]$

$z^2 - \frac{1}{2}z + 2 = 0 \Rightarrow z_{1,2} = 0.25 \pm j1.3919$

modes: $(1.4142)^n \angle \pm 1.3931n$ (rad) — unstable

(b) BIBO Stable

(c) $z^3 - 1.88z^2 + 0.99 = 0$ CE

$z_{1,2} = 1.2541 \pm j0.0546$, $z_3 = -0.6282$

$z_{1,2} = 1.2553 \angle \pm 0.0435$

modes: $(1.2553)^n \angle \pm 0.0435n$ (rad), $(-0.6282)^n$

Unstable

(d) $z^3 - z^2 + 2z - 3 = 0$; roots: $1.5335 \angle \pm 1.6608$ (rad), $1.2757$

unstable

(e) $z^2 - 4z + 1 = 0$; roots: $3.7321, 0.2679$

UNSTABLE.

# PROBLEM 10.25

(i)(a) $z - 0.9 = 0$; $z = 0.9$; modes: $(0.9)^n$

(b) $y_c[n] = \underline{C(0.9)^n}$

(ii)(a) $z^2 + 1.5z - 1 = 0$; $z_1 = -2$, $z_2 = 0.5$ modes: $(-2)^n, (0.5)^n$

(b) $y_c[n] = C_1(-2)^n + C_2(0.5)^n$

(iii)(a) $z^2 - 2z + 1 = 0$; roots: $z_1 = z_2 = 1$, modes: $(1)^n, (1)^n$

(b) $y_c[n] = C_1 + C_2 n$

(iv)(a) $z^2 - 1.7z + 0.72 = 0$; roots: $0.9, 0.8$; modes: $(0.9)^n, (0.8)^n$

(b) $y_c[n] = C_1(0.9)^n + C_2(0.8)^n$

(v)(a) $z^3 - 1 = 0$; roots: $1, 1\angle \pm 2\pi/3\,(\text{rad})$; modes: $(1)^n, 1\angle\frac{2\pi n}{3}, 1\angle\frac{-2\pi n}{3}$

(b) $y_c[n] = C_1 + C_2 e^{j2\pi n/3} + C_2^* e^{-j2\pi n/3}$

(vi)(a) $(z - 0.9)^3$, roots: $0.9, 0.9, 0.9$; modes: $(0.9)^n, (0.9)^n, (0.9)^n$

(b) $y_c[n] = C_1(0.9)^n + C_2 n(0.9)^n + C_3 n^2(0.9)^n$

(vii)(a) $(z - 0.9)(z - 1.2)(z + 0.85) = 0$; roots: $0.9, 1.2, -0.85$

modes: $(0.9)^n, (1.2)^n, (-0.85)^n$

(b) $y_c[n] = C_1(0.9)^n + C_2(1.2)^n + C_3(-0.85)^n$

$= C_1(0.9)^n + C_2(1.2)^n + C_3(0.85)^n \cos(\pi n)$

## PROBLEM 10.26

(i) stable, $(0.9)^n$ approaches zero as $n$ increases.

(ii) unstable: mode: $(-2)^n$ grows in magnitude without bound

(iii) unstable: $C_2 n$ increases without bound.

(iv) stable: $(0.9)^n$ and $(0.8)^n$ approach zero as $n$ increases

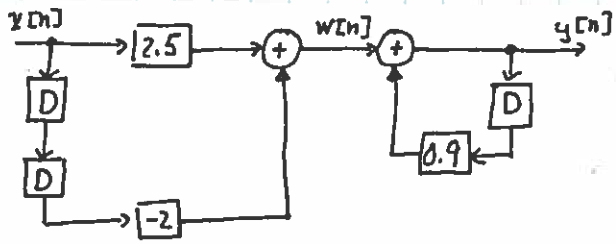
(v) not stable: $e^{j2\pi n/3}$ and $e^{-j2\pi n/3}$ have constant magnitude and rotating phase

(vi) stable: $(0.9)^n$ approaches zero for large values of $n$.

(vii) unstable. $(1.2)^n$ increases without bound.

## PROBLEM 10.27

(a)




(b) $y[-1]=0$ , $x[-1]=0$

$y[0]=2.5x[0]-2x[-2]=\underline{2.5}=h[0]$ , $x[0]=1$

$y[1]=0.9x[0]+2.5x[1]-2x[-1]=\underline{2.25}=h[1]$

$y[2]=0.9y[1]+2.5x[2]-2x[0]=2.025-2=\underline{0.025}=h[2]$

$y[3]=0.9y[2]=0.9(0.025)=\underline{0.0225=h[3]}$

$y[4]=0.9(0.0225)=\underline{0.02025}=h[4]$

(c) $w[0]=2.5(1)=2.5$ $\therefore$ $y[0]=2.5$

$w[1]=0$ $\quad y[1]=0.9(2.5)=2.25$

$w[2]=-2$ $\quad y[2]=0.9(2.25)-2=0.025$

$w[n]=0, n\geq 3$ $\quad y[3]=0.9(0.025)=0.0225$

$y[4]=0.9(0.0225)=0.02025$

(d) $y[n]=h[n+2]-3h[n]+2h[n-1]$

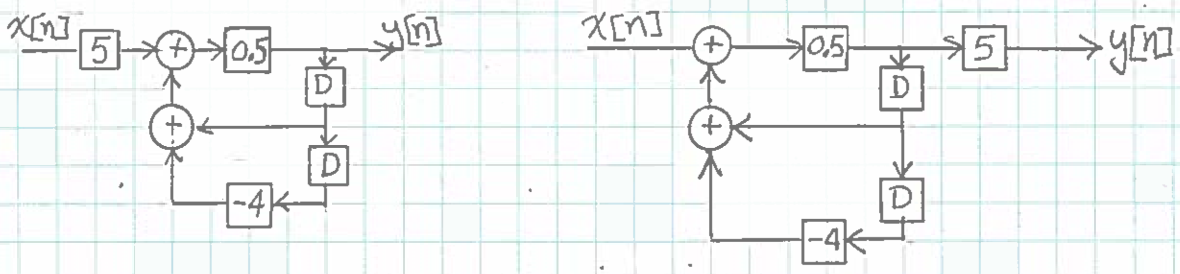
(e) $y[-3]=h[-1]-3h[-3]+2h[-4]=0$

$y[-1]=h[1]-3h[-1]+2h[-2]=2.25-0+0=2.25$

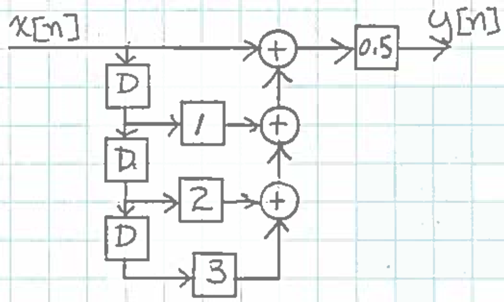
$y[1]=h[3]-3h[1]+2h[0]=0.025-3(2.25)+2.5=\underline{-4.225}$

## PROBLEM 10.28

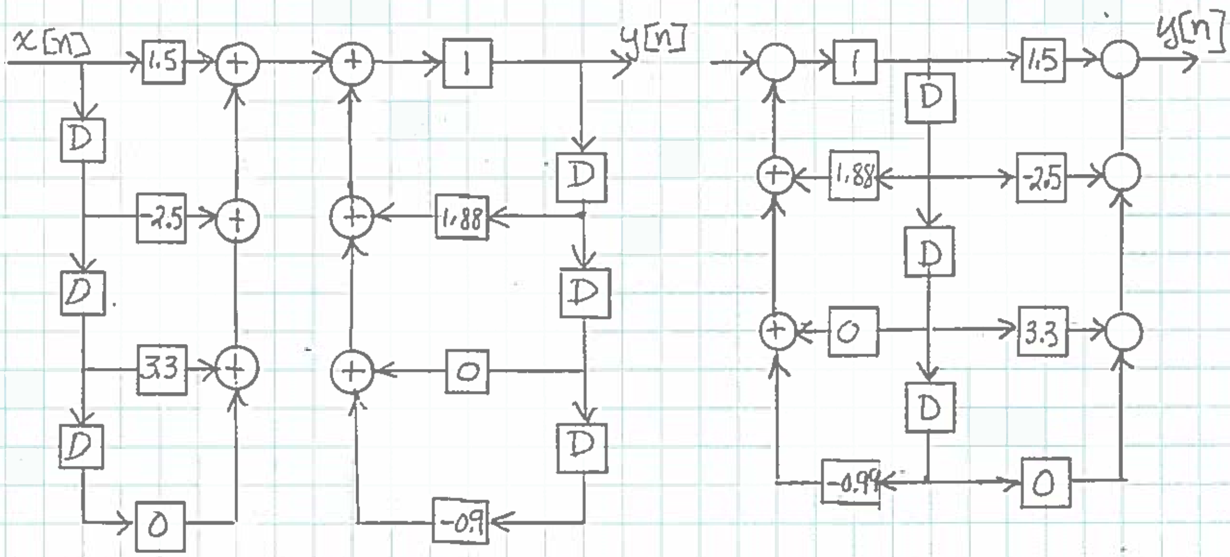
(a)




(b)

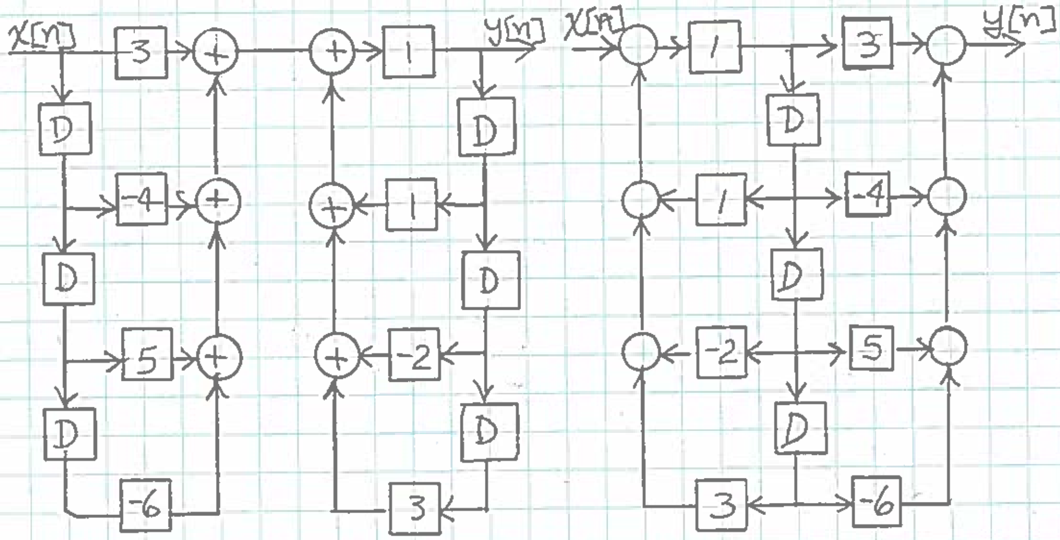



(c)

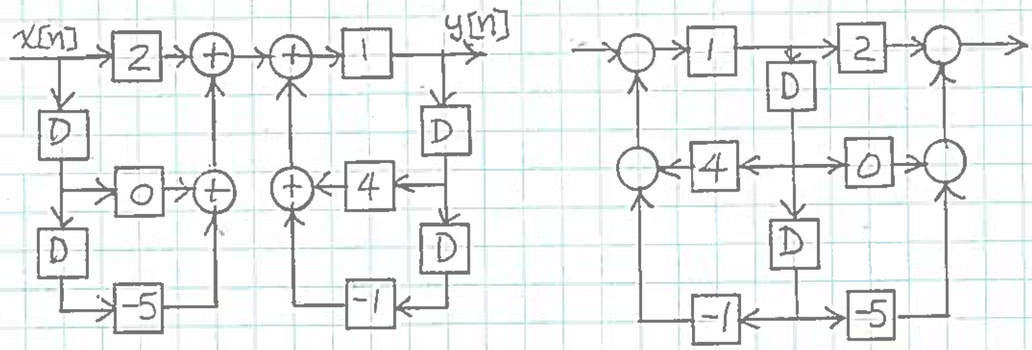


## PROBLEM 10.28 (continued)

(d)

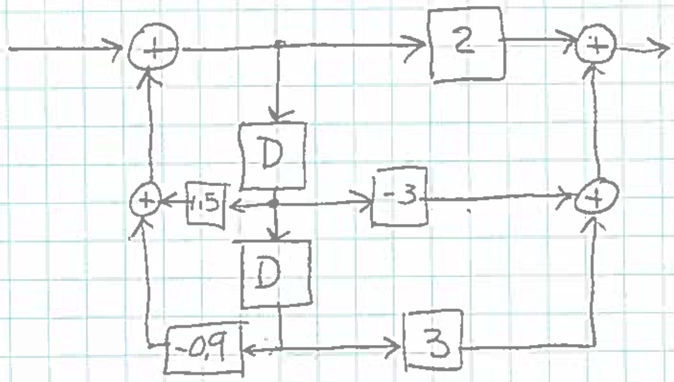



(e)



## PROBLEM 10.29

(a) Redraw the system diagram




(b) This can now be recognized as a Form II block diagram with: $\frac{1}{a_0} = 1$, $-a_1 = 1.5$, $-a_2 = -0.9$, $b_0 = 2$, $b_1 = -3$, and $b_2 = 3$

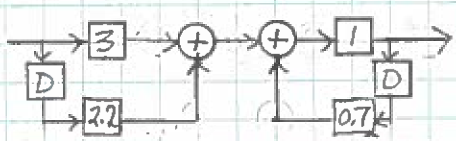
$$\therefore \; y[n] - 1.5y[n-1] + 0.9y[n-2] = 2x[n] - 3x[n-1] + 3x[n-2]$$

is the difference equation:

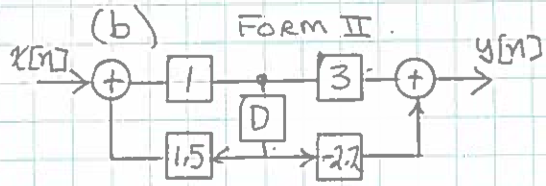
## PROBLEM 10.30

$$y[n] - 0.7y[n-1] = 3x[n] - 2.2x[n-1]$$

(a) Form I




(b) Form II




(c) $y[0] = 0.7y[-1] + 3x[0] - 2.2x[-1] = 3$

$$z - 0.7 = 0 \Rightarrow y_c[n] = C[0.7]^n$$

$$y_p[n] = P(0.8)^n \Rightarrow P(0.8)^n - 0.7(0.8)^{n-1} = 3(0.8)^n - 2.2(0.8)^{n-1}$$

$$P\cancel{(0.8)^n} - \frac{0.7}{0.8}P\cancel{(0.8)^n} = 3\cancel{(0.8)^n} - \frac{2.2}{0.8}\cancel{(0.8)^n}$$

$$P = \frac{3 - 2.2/0.8}{1 - 0.7/0.8} = 2 \Rightarrow y[n] = C(0.7)^n + 2(0.8)^n$$

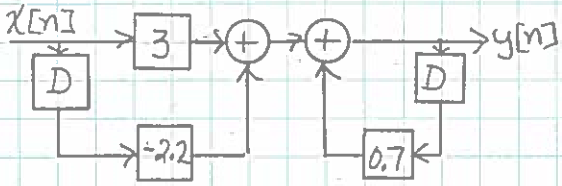
$$y[0] = 3 = C + 2 \Rightarrow C = 1$$

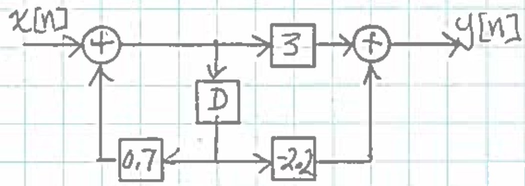
$$\therefore \; y[n] = (0.7)^n + 2(0.8)^n$$

(d)
```
>>y(1)=3
>>for n=1:5
     y[n+1] = 0.7*y[n] + 3*((0.8)^n) - 2.2*((0.8)^(n-1));
  end
>> y
```

## PROBLEM 10.31

(a) $y[n] - 0.7y[n-1] = 3x[n] - 2.2x[n-1]$ ; $y[0] = 0$

(b)



(c)



(d) $x[n] = (0.8)^n u[n]$

(e) See solution for Problem 10.30(c).

(f) See solution for Problem 10.30(d)

## PROBLEM 10.32

(a) $y[n] - 0.7y[n-1] = 3x[n] - 2.2x[n-1]$

(b) $y_p[n] = P \Rightarrow P - 0.7P = 3 - 2.2 \Rightarrow P = \dfrac{0.8}{0.3} = 2.667$

(c) (This requires a look ahead to section 10.7)

From (10.80): $H(z) = \dfrac{b_0 z + b_1}{a_0 z + a_1} = \dfrac{3z - 2.2}{z - 0.7}$

(d) $H(1) = \dfrac{3-2.2}{1-0.7} = 2.667 \Rightarrow y_{ss}[n] = 2.667$

(e)
```
>> y[1] = 0
>> for n=1:25
      x(n) = 1
   end
>> for n = 2:25
      y(n) = 0.7*y(n-1) + 3*x(n) - 2.2*x(n-1);
   end
>> y
```

## PROBLEM 10.33

$y[n] = a(x[n] + b\,y[n-1]) \Rightarrow y[n] - ab\,y[n-1] = a\,x[n]$

From (10.80): $H(z) = \dfrac{az}{z - ab} \Rightarrow a = 0.1;\ b = 9.$

(This problem requires a look ahead to Section 10.7)

## PROBLEM 10.34

(a) $y[n] - 0.7\,y[n-1] = x[n]$

From (10.80): $H(z) = \dfrac{z}{z - 0.7}$

(b) $x[n] = \cos(n)u[n] \Rightarrow \cos(n\Omega)u[n],\ \Omega = 1$

From (10.83) $\cos(\Omega n) \longrightarrow |H(e^{j\Omega})|\cos(\Omega n + \theta_H)$

$e^{j1} = \cos 1 + j\sin 1 = 0.54 + j0.84$

$H(e^{j1}) = \dfrac{0.54 + j0.84}{0.54 + j0.84 - 0.7} = \dfrac{1\angle 57.3^\circ}{0.855\angle 100.8^\circ} = 1.169\angle -43.5^\circ$

$\Rightarrow y_{ss}[n] = 1.169\cos(n - 43.5^\circ)$

(c)
```
>> n = [1  0];
>> d = [1  -0.7];
>> z = exp(j);
>> H = polyval(n,z)/polyval(d,z)
>> ymag = abs(H)
>> yph  = angle(H)*180/pi
```

(d) $y_{ss}[n] - 0.7\,y_{ss}[n-1] = \cos(n)u[n]$

$1.169\cos(n - 43.5^\circ) - 0.7(1.169)\cos(n - 57.3^\circ - 43.5^\circ)$

$= 1.169\cos(n - 43.5^\circ) - 0.818\cos(n - 100.8^\circ)$

$= 1.169[\cos(n)\cos(43.5^\circ) + \sin(n)\sin(43.5^\circ)]$

$\quad -0.818[\cos(n)\cos(+100.8) + \sin(n)\sin(100.8)]$

$= 0.848\cos(n) + 0.805\sin(n) + 0.153\cos(n) - 0.804\sin(n)$

$= 1.001\cos(n) + 0.001\sin(n) \simeq \cos(n)$ ✓

## PROBLEM 10.35

(a) From (10.80): $H(z) = \dfrac{z^2}{z^2 - 1.7z + 0.72}$

(b) $H(1) = \dfrac{1}{1 - 1.7 + 0.72} = 50$; $\therefore y_{ss}[n] = (1)H[1] = 50$.

(c) $x[n] = \cos(n)u[n] \Rightarrow \Omega = 1$

From (10.83) $\cos(\Omega n) \longrightarrow (1)|H(e^{j\Omega})|\cos(\Omega n + \theta_H)$

$e^{j\Omega} = e^{j1} = 1\angle 57.3^\circ = 0.540 + j0.841$

$$H(e^{j1}) = \frac{(1\angle 57.3^\circ)^2}{(1\angle 57.3^\circ)^2 - 1.7(1\angle 57.3^\circ) + 0.72} = -0.336 - j1.195$$

$H(e^{j1}) = 1.241\angle -105.7^\circ$

$y_{ss}[n] = 1.241\cos(n - 105.7^\circ)$

(d)
```
>> n=[1 0 0]; d=[1 -1.7 0.72];
>> H = polyval(n,1)/polyval(d,1)
```

(e)(b) $y_{ss}[n] - 1.7y_{ss}[n-1] + 0.72y_{ss}[n-2] = 50 - (1.7)(50) + 0.72(50) = 1$ ✓

(c) $1.241\cos(n - 105.7) - 1.7(1.241)\cos(n - 105.7^\circ - 57.3^\circ)$
$+ 0.72(1.241)\cos(n - 105.7^\circ - 114.6^\circ) = \cos(n)$

$1.241[\cos(n)\cos(105.7^\circ) + \sin(n)\sin(105.7^\circ)]$
$-1.7(1.241)[\cos(n)\cos(163^\circ) + \sin(n)\sin(163^\circ)]$
$+0.72(1.241)[\cos(n)\cos(220.3^\circ) + \sin(n)\sin(220.3^\circ)]$
$= -0.336\cos(n) + 1.195\sin(n) + 2.018\cos(n) - 0.617\sin(n)$
$- 0.681\cos(n) - 0.578\sin(n)$
$= 1.001\cos(n) \simeq \cos(n)$ ✓

# PROBLEM 10.36

$$H(z) = \frac{0.1z}{z^2 - 1.8z + 0.81}$$

(a) (10.48) & (10.80): $y[n] - 1.8y[n-1] + 0.81y[n-2] = 0.1x[n-1]$

(b) $(z-0.9)^2 = 0$ $\therefore$ modes: $(0.9)^n$, $n(0.9)^n$

(c) $y_c[n] = C_1(0.9)^n + C_2 n(0.9)^n$

(d) $x[n] = \cos(0.2n) = \cos \Omega n$ , $\Omega = 0.2 \Rightarrow 11.46^\circ$

(10.83) $\cos \Omega n \longrightarrow (1)|H(e^{j\Omega})| \cos(\Omega n + \theta_H)$

$$e^{j\Omega}\Big|_{\Omega=2} = e^{j0.2} = 1\underline{/11.46^\circ} = 0.9801 + j0.199$$

$$H(e^{j0.2}) = \frac{0.1(1\underline{/11.46^\circ})}{(1\underline{/11.46^\circ}) - 1.8(0.980 + j0.199) + 0.81} = 2.208\underline{/-125.3^\circ}$$

$\therefore y_p[n] = 2.208 \cos(0.2n - 125.3^\circ)$

(e)

```
n=[0 .1  0];
d=[1 -1.8 .81];
z=exp(j*.2)
h=polyval(n,z)/polyval(d,z);
yabs=abs(h)
yphase=angle(h)*180/pi
```

(f) $2.208 \cos(0.2n - 125.3^\circ) - 1.8(2.208)\cos(0.2n - 125.3^\circ - 11.5^\circ)$

$+0.81(2.208)\cos(0.2n - 125.3^\circ - 23^\circ) = 0.1\cos(0.2n - 11.46^\circ)$

$\therefore 0.101 \cos 0.2n + 0.023 \sin 0.2n \cong 0.098 \cos 0.2n + 0.020 \sin 0.2n$

(g) $x[n] = \cos[(2\pi + 0.2)n]$ $\therefore \Omega = 2\pi + 0.2$

$$e^{j\Omega} = e^{j2\pi}e^{j0.2} = e^{j0.2}$$

$\therefore$ answer same as (d).

(i) $H(e^{j\Omega})$ is periodic in $\Omega$ with period $2\pi$.