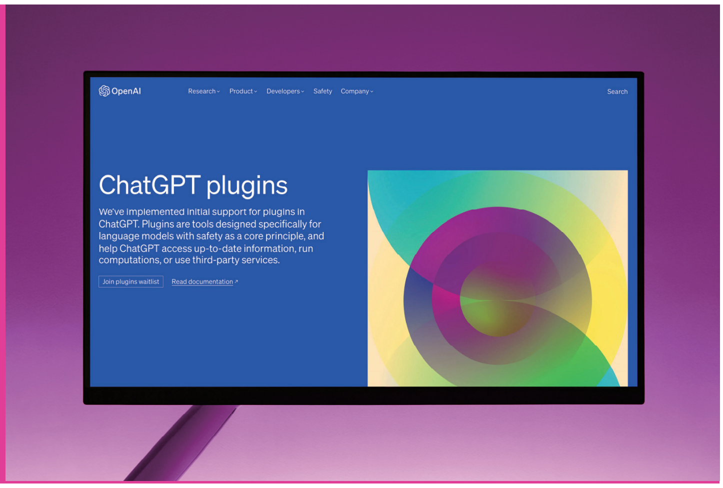
DISCRETE-TIME LINEAR TIME-INVARIANT SYSTEMS

CHAPTER 10



10.1 IMPULSE REPRESENTATION OF DISCRETE-TIME SIGNALS

- IMPULSE REPRESENTATION OF DISCRETE-TIME SIGNALS

$$\delta[n-n_0]=\begin{cases}1, & n=n_0\\ 0, & n\neq n_0\end{cases}.$$

$$x[n]\delta[n-n_0]=x[n_0]\delta[n-n_0].$$

- We define the signals

$$x_{-1}[n]=x[n]\delta[n+1]=x[-1]\delta[n+1]=\begin{cases}x[-1], & n=-1\\ 0, & n\neq -1\end{cases}.$$

$$x_0[n]=x[n]\delta[n]=x[0]\delta[n]=\begin{cases}x[0], & n=0\\ 0, & n\neq 0\end{cases};$$

$$x_1[n]=x[n]\delta[n-1]=x[1]\delta[n-1]=\begin{cases}x[1], & n=1\\ 0, & n\neq 1\end{cases}.$$

10.2 Convolution for discrete-time systems

- Property of convolution sum:

$$\delta[n]*g[n-n_0] = \delta[n-n_0]*g[n] = g[n-n_0].$$

$$\delta[n]g[n-n_0] = g[-n_0]\delta[n]$$

$$\delta[n-n_0]g[n] = g[n_0]\delta[n-n_0],$$

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k].$$

Representation of a signal with discrete-time impulse functions.

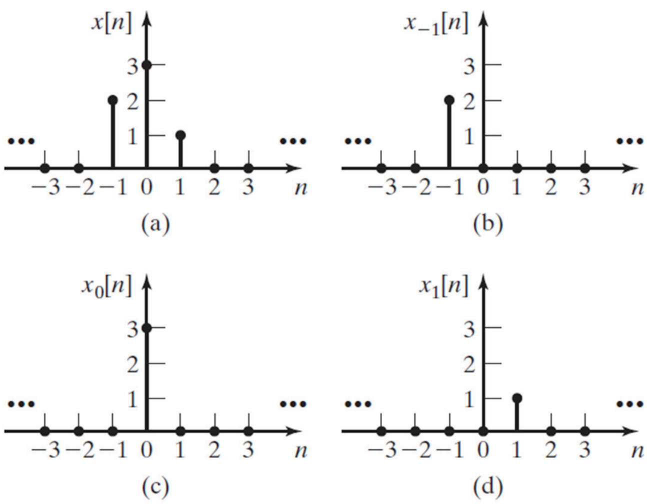


Figure 10.1 Representation of a signal with discrete-time impulse functions.

Convolution sum for DT signals

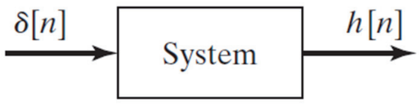
- 

Figure 10.2 Impulse response of a system.

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = x[n]*h[n]$$
$$= \sum_{k=-\infty}^{\infty} h[k]x[n-k] = h[n]*x[n].$$

- To illustrate a property of the convolution sum, we calculate the output at $n = 0$.

$$y[0] = \cdots + x[-2]h[2] + x[-1]h[1] + x[0]h[0] + x[1]h[-1] + x[2]h[-2] + \cdots.$$

EXAMPLE 10.1 A Finite Impulse Response System

- The system difference equation directly from Figure 10.3 is

$$y[n] = (x[n] + x[n-1] + x[n-2])/3.$$

$$h[n] = (\delta[n] + \delta[n-1] + \delta[n-2])/3.$$

$$h[0] = (\delta[n] + \delta[n-1] + \delta[n-2])/3|_{n=0} = (1 + 0 + 0)/3 = 1/3;$$
$$h[1] = (\delta[n] + \delta[n-1] + \delta[n-2])/3|_{n=1} = (0 + 1 + 0)/3 = 1/3;$$
$$h[2] = (\delta[n] + \delta[n-1] + \delta[n-2])/3|_{n=2} = (0 + 0 + 1)/3 = 1/3;$$
$$h[n] = 0, \text{ all other } n.$$

This is a finite impulse response (FIR) system; that is, the impulse response contains a finite number of nonzero terms.

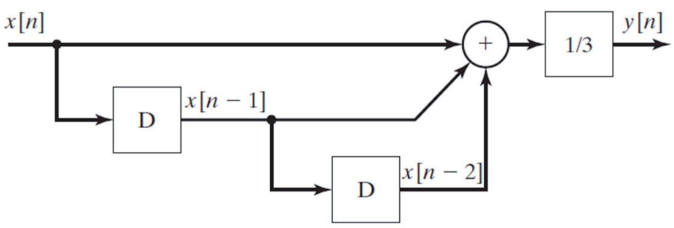
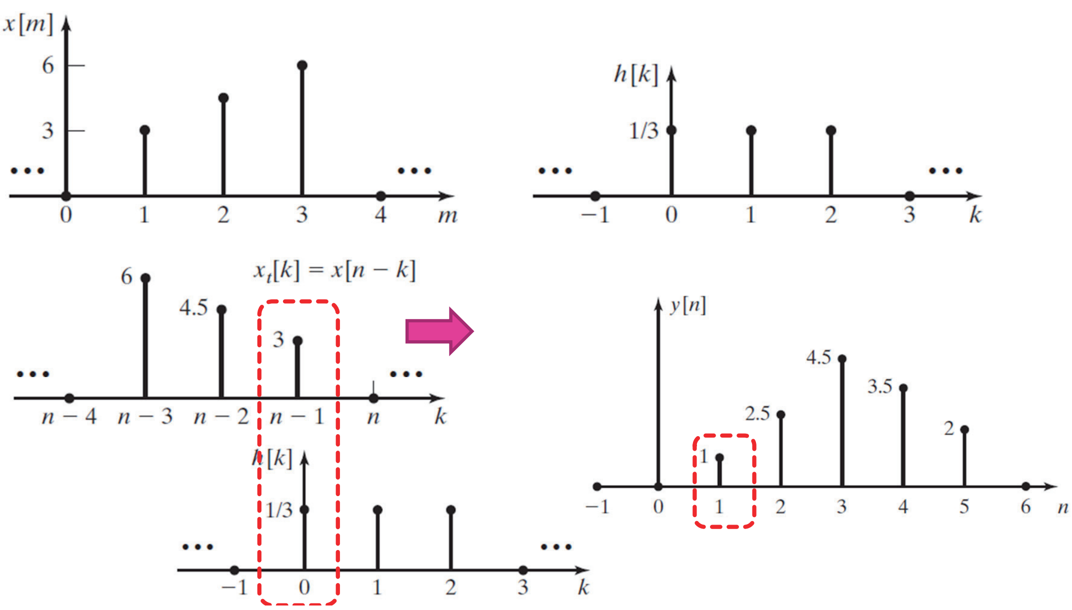


Figure 10.3

EXAMPLE 10.2 （必考題）

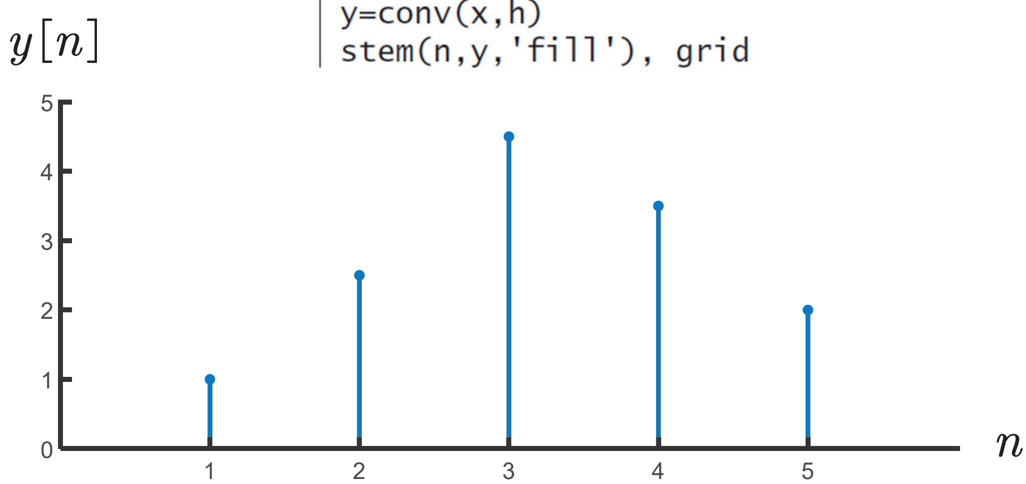
System Response by Convolution for An LTI System

- Find the convolution sum $x[n] * h[n]$. $y[n] = \sum_{k=-\infty}^{\infty} x[n-k]h[k]$.



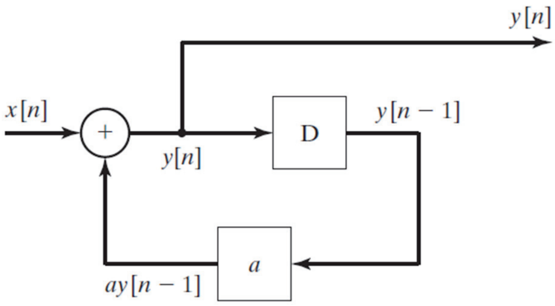
MATLAB® code of convolution sum

```
n=1:5;
x=[3 4.5 6];
h=[1/3 1/3 1/3];
y=conv(x,h)
stem(n,y,'fill'), grid
```



EXAMPLE 10.3
Calculation of the impulse response of a discrete system

$$y[n] = ay[n-1] + x[n].$$



$$h[n] = ah[n-1] + \delta[n]$$

$$\begin{cases} h[0] = a \times 0 + \delta[0] = 1 \\ h[1] = ah[0] + \delta[1] = a \\ \vdots \\ h[n] = a^n,\ n \geq 0 \end{cases}$$

$$h[n] = a^n u[n]$$

- The unit impulse response consists of an unbounded number of terms; this system is called an infinite impulse response (IIR) system

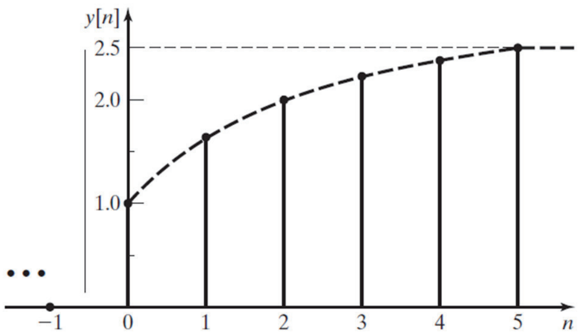
EXAMPLE 10.4
Step response of a discrete system with $h[n] = 0.6^n u[n]$

- Find the unit step response of this system, with $x[n] = u[n]$.

$$y[n] = \sum_{k=-\infty}^{\infty} x[n-k]h[k] = \sum_{k=-\infty}^{\infty} u[n-k](0.6)^k u[k] = \sum_{k=0}^{n} (0.6)^k,$$

$$\sum_{k=0}^{n} a^k = \frac{1-a^{n+1}}{1-a}.$$

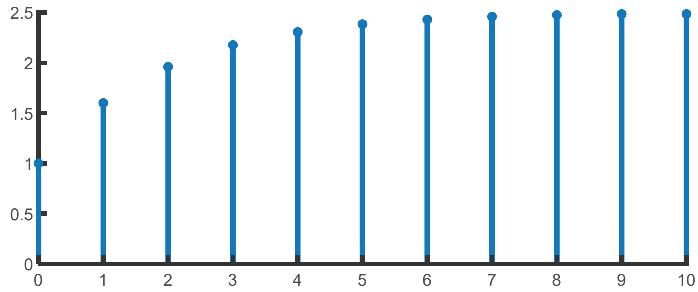
$$y[n] = \sum_{k=0}^{n} (0.6)^k = \frac{1-(0.6)^{n+1}}{1-0.6} = 2.5[1-(0.6)^{n+1}], \quad n \geqq 0.$$



MATLAB® EXAMPLE CODE

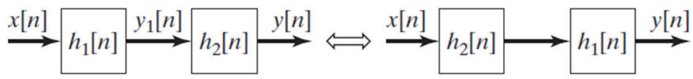
- Unit step response

```
% This MATLAB program performs the convolution operation of
% Example 10.4.
% Establish the input unit-step function.
x=ones (1,11);
% Compute the system impulse response.
for k = 1:10;
h(k) = 0.6^(k-1);
end
% Convolve the input with the impulse response and plot.
c=conv (x,h);
for k=1:11;
    n(k)=k-1;
    y(k)=c(k);
end
 [n' y']
 stem(n, y, 'fill')
```

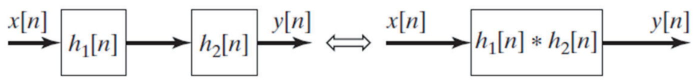


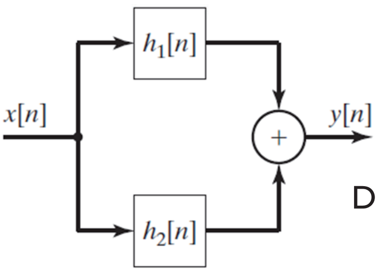
Properties of convolution Sum

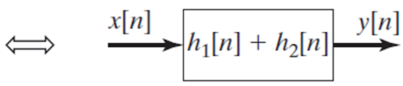
a) Commutative property
b) Associative property
c) Distributive property



(a)







Distributive property

10.3 Properties of Discrete-time LTI Systems

- **Memory**
  - A memoryless LTI system is then a pure gain, described by $y[n] = Kx[n]$.
- **Invertibility**

$$h[n]*h_i[n] = \delta[n],$$

  - We do not present a procedure for finding the impulse response given $h[n]$. This problem can be solved with the use of the z-transform of Chapter 11.
- **Causality**

$$y[n_1] = T(x[n]), \quad n \leqq n_1,$$

  - As an additional point, a signal that is zero for is called a *causal signal*
  - For a causal LTI discrete-time system, $h[n] = 0$ for $t < 0$.

10.3 Properties of Discrete-time LTI Systems

- **Stability**
  - $h[n]$ is said to be *absolutely summable*. ⇔ $\sum_{k=-\infty}^{\infty} |h[k]| < \infty.$

EXAMPLE 10.6
Stability of an LTI discrete system with $h[n] = \left(\frac{1}{2}\right)^n u[n]$

$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n = \frac{1}{1 - 1/2} = 2.$$

10.4 Difference-Equation Models

- A first-order discrete-time system model

$$y[n] = ay[n-1] + bx[n].$$

- The general form of an Nth-order linear difference equation with constant coefficients is,

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k], \quad a_0 \neq 0.$$

- It can be shown by these procedures that this equation is both linear and time invariant.

EXAMPLE 10.11
Stability of a discrete system

- Suppose that a causal system is described by the difference equation

$$y[n] - 1.25y[n-1] + 0.375y[n-2] = x[n].$$

特性方程式 $1 - 1.25z^{-1} + 0.375z^{-2} = 0,$

$$z^2 - 1.25z + 0.375 = 0, \quad \rightarrow \quad z = 0.75,\ 0.5$$

- The natural response is given by

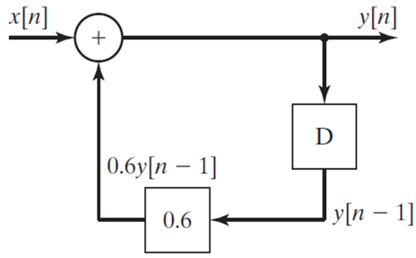
$$y_c[n] = C_1(0.75)^n + C_2(0.5)^n.$$

- This function approaches zero as n approaches infinity. It's a stable system.

All roots satisfy $|z| < 1$, then the system is stable.

EXAMPLE 10.12
Simulation diagram for a discrete system

- Considered a discrete-time system described by the difference equation $y[n] - 0.6y[n-1] = x[n]$.



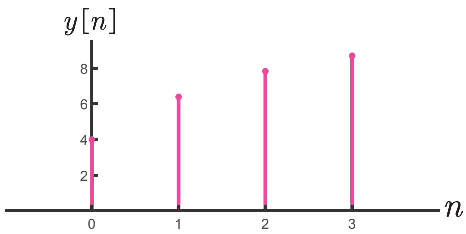
The transfer function

$$H(z) = \frac{1}{1 - 0.6z^{-1}}$$

$$= \frac{z}{z - 0.6}$$

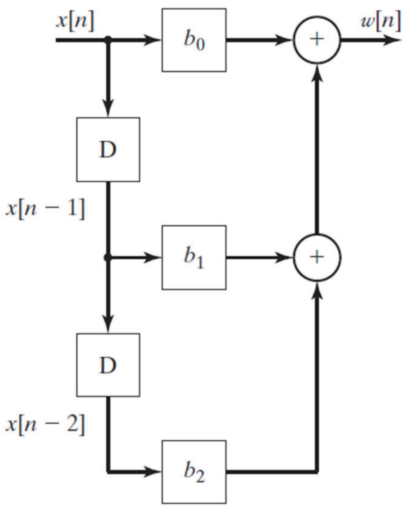
MATLAB program with $x[n] = 4$.

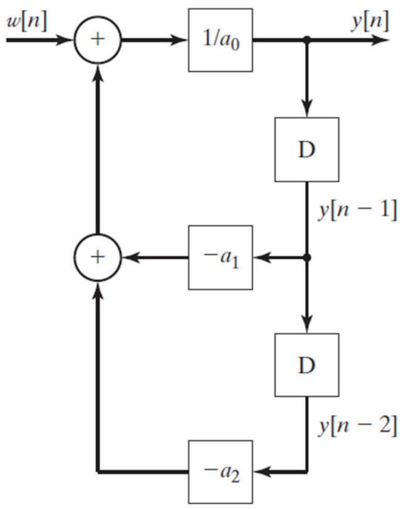
```
ynminus1=0;
for n=0:3
   yn=0.6*ynminus1+4
   ynminus1=yn;
end
result: yn=4 6.4 7.84 8.704
```



BLOCK DIAGRAM (TYPE I)

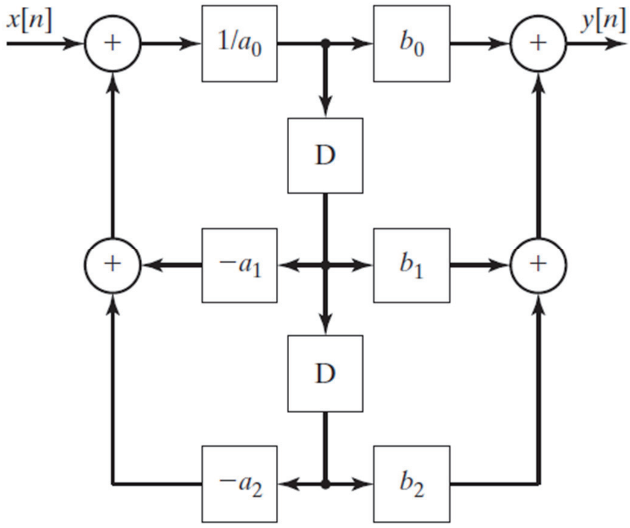
$$a_0y[n] + a_1y[n-1] + a_2y[n-2] = b_0x[n] + b_1x[n-1] + b_2x[n-2].$$





BLOCK DIAGRAM (TYPE II)

$$a_0 y[n] + a_1 y[n-1] + a_2 y[n-2] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2].$$



EXAMPLE 10.14
Transfer function for a discrete system

- The α-filter

$$y[n] - (1-\alpha)y[n-1] = \alpha x[n].$$

$$H(z) = \frac{\alpha}{1-(1-\alpha)z^{-1}} = \frac{\alpha z}{z-(1-\alpha)}.$$

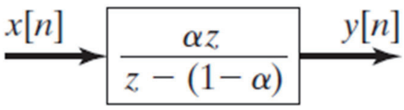
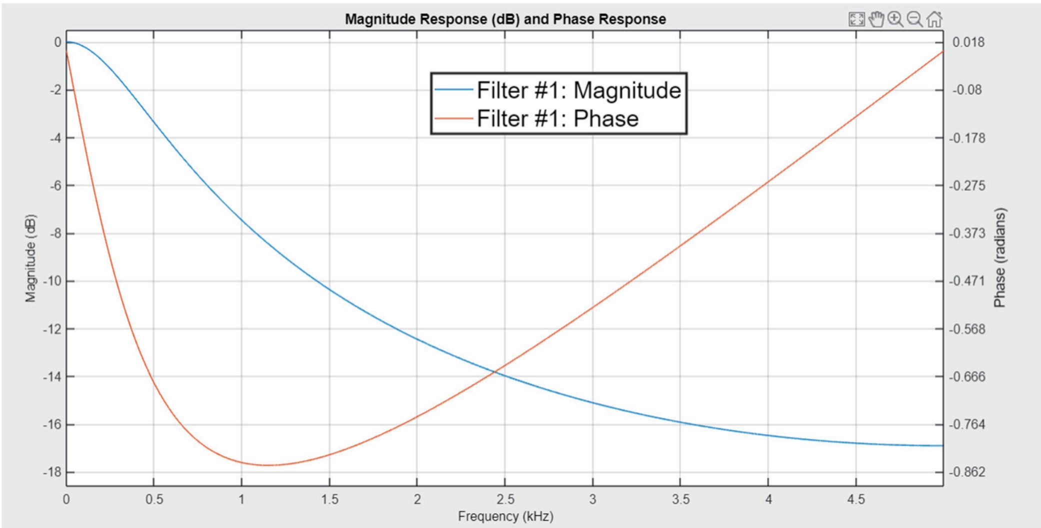


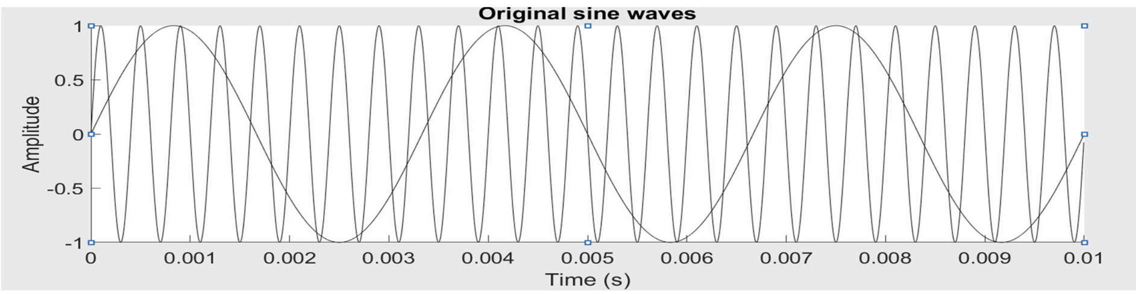
Figure 10.21 α-Filter.

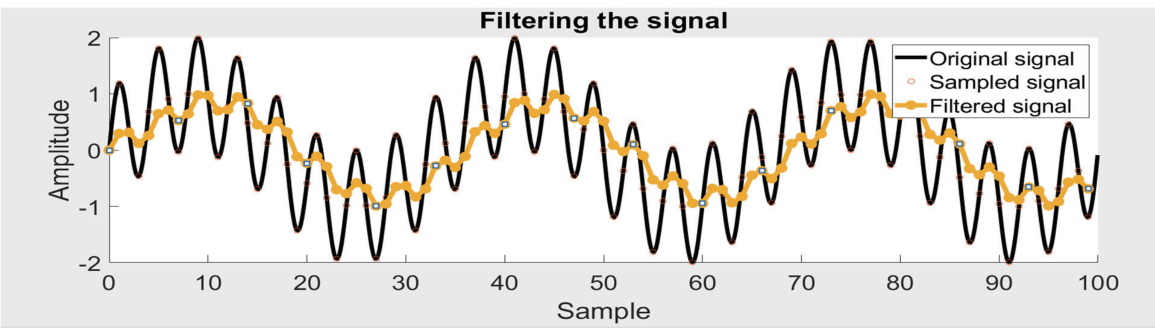
Frequency response of an α −filter

- Alpha-filter

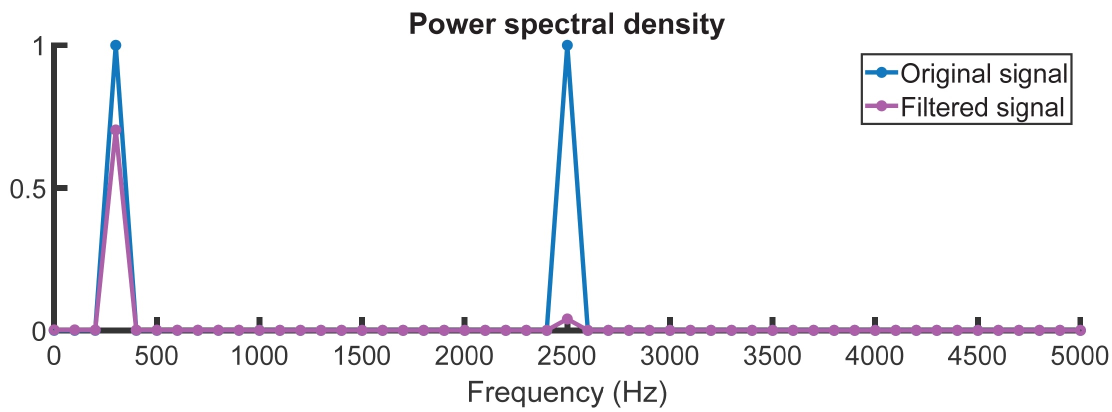


α −filter (Input waves and filtered output wave)





Spectra of Input and Filtered Waves of the α-filter



LTI System Input-Output Functions

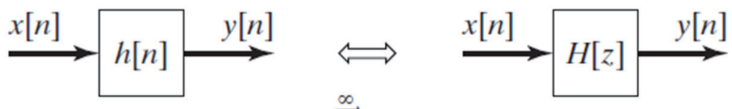
- LTI systems

TABLE 10.1 Input–Output Functions for an LTI System

$$H(z) = \sum_{k=-\infty}^{\infty} h[k]z^{-k}$$

$$Xz_1^n \rightarrow XH(z_1)z_1^n, \quad X = |X|e^{j\phi}$$

$$|X|\cos(\Omega_1 n + \phi) \rightarrow |X||H(e^{j\Omega_1})|\cos(\Omega_1 n + \phi + \theta_H)$$



$$H[z] = \sum_{k=-\infty}^{\infty} h[k]z^{-k}$$

Figure 10.22 LTI system.

CHAPTER 10

- THE END -