Q1. (a) 依照題目的波形得到 $\frac{3}{2V}x(-t)-\frac{1}{2}$ 或 $\frac{-3}{2V}x(t)-\frac{1}{2}$

(b) 使用 $A=\frac{-3}{2V}, B=-\frac{1}{2}$ (如果使用 $A=\frac{3}{2V}, B=-\frac{1}{2}$ 因為時間反轉，C_k 要取共軛)。

$C_{0y}=C_{0x}+B=-\frac{1}{2}$ 且 $C_{ky}=AC_{kx}=-\frac{3}{2V}\times\frac{2V}{k\pi}e^{-j\pi/2}=\frac{3}{k\pi}e^{j\frac{\pi}{2}}$ (k:odd, $k\neq 0$).

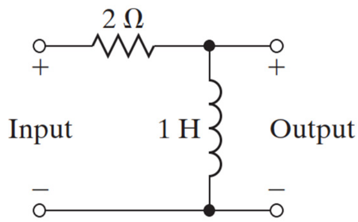
The Fourier series is expressed by

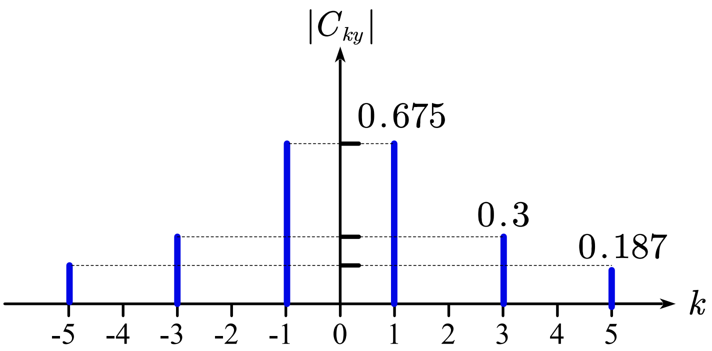
$$y(t)=\frac{-1}{2}+\sum_{\substack{-\infty \\ k\ odd,\ k\neq 0}}^{\infty}\frac{3}{k\pi}e^{j\pi/2}e^{j2k}.$$

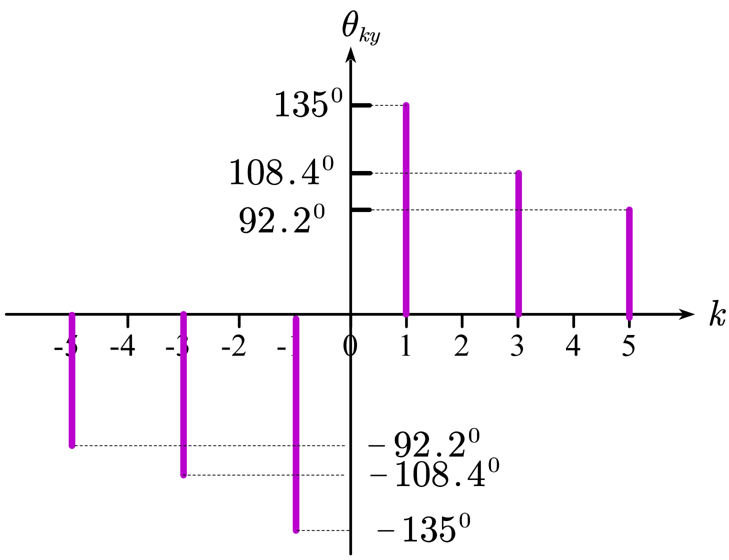
(c) 轉移函數 $H(j\omega)=\frac{j\omega}{2+j\omega}$，訊號基本頻率 $\omega_0=\frac{2\pi}{\pi}=2.$ $H(j2k)=\frac{j2k}{2+j2k}=\frac{jk}{1+jk}$

$C_{0y}=H(0)C_{0x}=0\times\left(-\frac{1}{2}\right)=0,\ k=0.$

$$C_{ky}=H(j2k)C_{kx}=\frac{3}{\pi\sqrt{1+k^2}}e^{j(180^o-\tan^{-1}(k))},\ k\ \text{odd},\ k\neq 0$$







Q2. $x(t)=1+2\cos\left(2\pi t+\frac{\pi}{4}\right)+6\sin\left(6\pi t-\frac{\pi}{4}\right)$

(a) $C_0=1, C_1=C_{-1}^*=e^{j\pi/4}, C_3=C_{-3}=3e^{-j3\pi/4}$

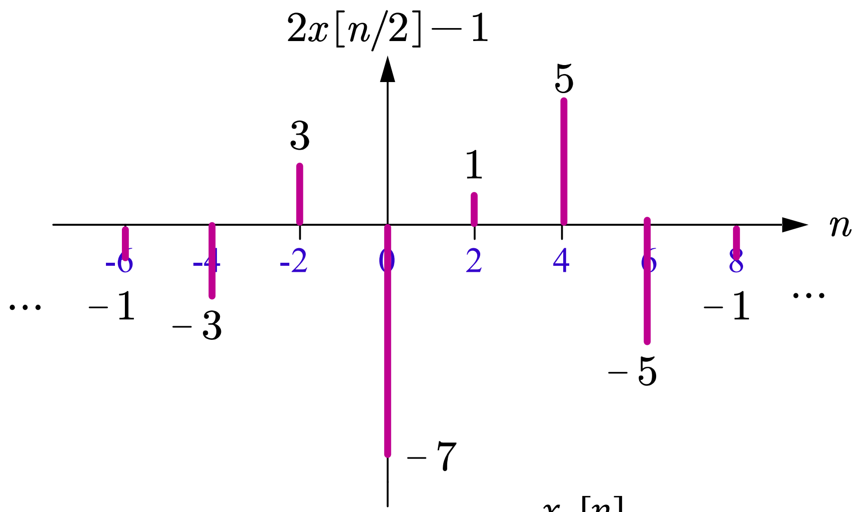
(b) $C_k=\frac{1}{T_0}\int_{-T_0/2}^{T_0/2}2\delta(t)dt=\frac{1}{2}\times 2=1$

(c) $C_0=1, C_k=\frac{j}{k\pi}, k\neq 0.$

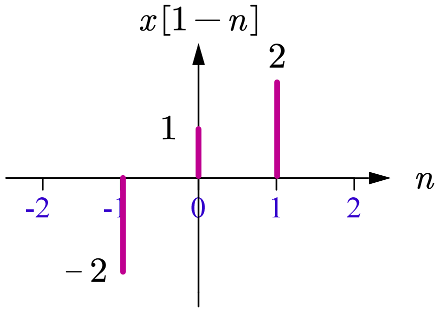
$$C_k=\frac{1}{2}\left[\frac{t}{-jk\pi}e^{-jk\pi t}\Big|_0^2-\frac{1}{(jk\pi)^2}e^{-jk\pi t}\Big|_0^2\right]=\frac{j}{k\pi},\ k\neq 0.$$

$$C_0=\frac{1}{2}\times\left(\frac{1}{2}\times 4\right)=1$$

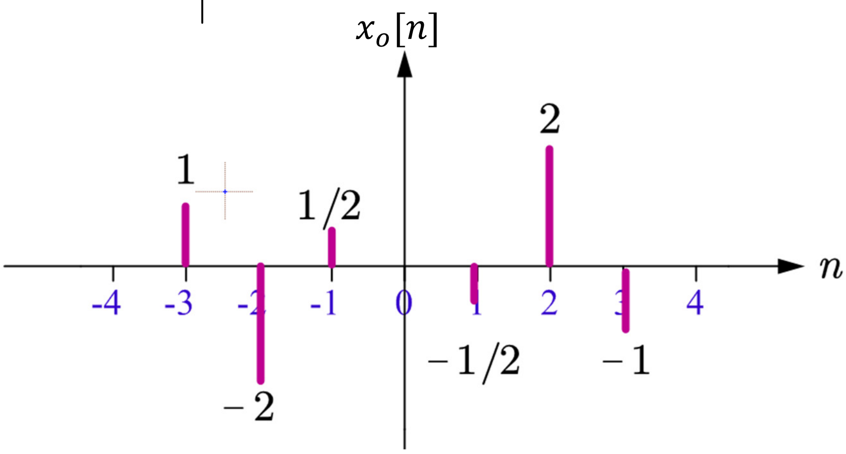
Q3. (a)



(b)



(c)



Q4. (a) $k=5, N=14$ (b) 84 (c) $\tau=-\frac{T}{\ln(0.5)}=0.289$

(a) $T_0=\dfrac{2\pi}{2\pi/7}=7,\ \dfrac{T_0}{T}=\dfrac{7}{5/2}=\dfrac{14}{5}$

$5T_0=14T,\ \therefore\ k=5,\ N=14$

(b) $\dfrac{T}{T_{01}}=\dfrac{\Omega_{01}}{2\pi}=\dfrac{1}{6},\ N_{01}=6$

$\dfrac{T}{T_{02}}=\dfrac{\Omega_{02}}{2\pi}=\dfrac{3}{7},\ N_{02}=7$

$\dfrac{T}{T_{03}}=\dfrac{\Omega_{03}}{2\pi}=\dfrac{5}{4},\ N_{03}=4$

$lcm(6,7,4)=84$ samples/period

(c) $e^{-nT/\tau}=(0.5)^n,\ \dfrac{-nT}{\tau}=n\ln(0.5),\ \ \tau=\dfrac{-T}{\ln(0.5)}=\dfrac{-1/5}{\ln(0.5)}=0.288539$

Q5. (a) time-invariant

$$y_d[n] = y[n]|_{x[n-n_0]} = \frac{1}{3}(x[n-n_0+1] + x[n-n_0] + x[n-n_0-1])$$

$$y[n-n_0] = \frac{1}{3}(x[n-n_0+1] + x[n-n_0] + x[n-n_0-1])$$

$$\because\ y_d[n] = y[n-n_0] \quad \therefore\ \text{time} - \text{invariant}$$

linear

Suppose $x_1[n] \longrightarrow y_1[n],\ x_2[n] \longrightarrow y_2[n]$

Test the input $a_1x_1[n] + a_2x_2[n]$

$$\longrightarrow \frac{1}{3}(a_1x_1[n+1] + a_2x_2[n+1] + a_1x_1[n] + a_2x_2[n] + a_1x_1[n-1] + a_2x_2[n-1])$$

$$= \frac{a_1}{3}(x_1[n+1] + x_1[n] + x_1[n-1]) + \frac{a_2}{3}(x_2[n+1] + x_2[n] + x_2[n-1])$$

$$= a_1y_1[n] + a_2y_2[n] \quad \therefore\ \text{linear}$$

注意上面箭頭不可以寫成「等號」

noncausal

$$y[n] = \frac{1}{3}(x[n+1] + x[n] + x[n-1])$$

The output at $n = n_0$ is related to the input at $n = n_0 + 1$.

$\because\ n_0 < n_0 + 1$ (只寫這個不等式零分) $\therefore$ noncausal

必須指出n_0, n_0+1是誰的

(b) $h[n] = \frac{1}{3}(\delta[n+1] + \delta[n] + \delta[n-1])$

$\because\ \delta[n] \longrightarrow h[n]$, Replace $x[n]$ with $\delta[n]$ in the system model.

We have $h[n] = \frac{1}{3}(\delta[n+1] + \delta[n] + \delta[n-1])$.

Q6. (a) $y[n] = x[n] + 0.5y[n-1]$ (b) $h[n] = (0.5)^n u[n]$

$$h[0] = \delta[0] + 0 = 1$$

$$h[1] = \delta[1] + 0.5h[0] = 0.5$$

$$h[2] = \delta[2] + 0.5h[1] = 0.5^2$$

$$\vdots$$

$$h[n] = \delta[n] + 0.5h[n-1] = 0.5^n,\ n \geq 0 = 0.5^n u[n]$$

(c) $\sum_{n=0}^{\infty}|h[n]| = 2 < \infty$ Stable

$$\sum_{n=0}^{\infty}|h[n]| = \sum_{n=0}^{\infty}|0.5^n| = \frac{1}{1-0.5} = 2 < \infty$$ It is BIBO stable.

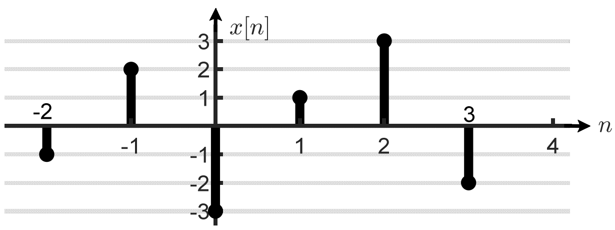
(d) $y[n] = (n+1)(0.5)^n u[n]$

$$y[n] = \sum_{k=0}^{n} x[k]x[n-k] = \sum_{k=0}^{n}(0.5)^k(0.5)^{n-k} = \sum_{k=0}^{n}(0.5)^n$$

$$= (n+1)(0.5)^n,\ n \geq 0.$$

(e)

$$y[n] = x[n] * x[n] = \sum_{k=-\infty}^{\infty} x[k]x[n-k]$$



使用圖解法