CHAPTER II

2.1 Each of the following can be answered using either set algebra or Venn diagram.

(a) Using set algebra,

$$AB(C \cup B) = (ABC) \cup (ABB) = (ABC) \cup (AB) = AB$$

Hence, $AB(C \cup B) \neq ABC$.

Using Venn diagram,

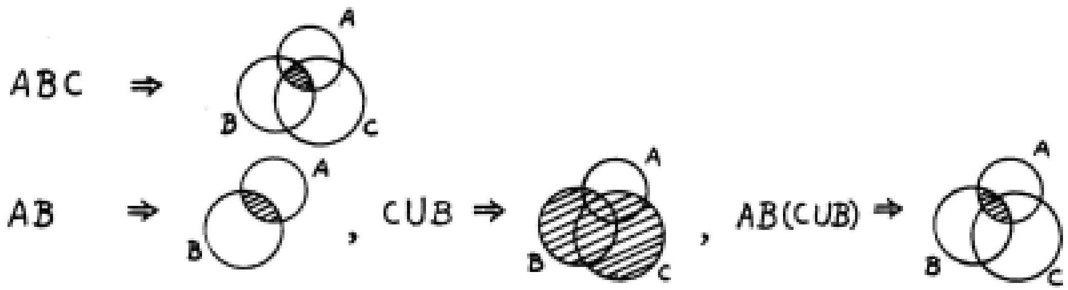


Figure 2.1

Hence, $ABC \neq AB(C \cup B)$ since the areas they occupy are not the same. Similar procedures apply to other parts. We have

(a) Incorrect (b) Correct (c) Correct
(d) Correct (e) Correct (f) Correct

2.2 $A \cup B \cup C = A + \overline{A}(B \cup C) = A + \overline{A}(B + \overline{B}C)$

$$= A + \overline{A}B + \bar{A}\bar{B}C$$

2.3 (a) Let $B_k = \bigcup_{j=1}^{k} A_j,\ k = 1, 2, \ldots, n.$

Then $B_k = B_{k-1} \bigcup A_k$.

From the third relation in Eqs. (2.10), we have

$$\overline{B}_k = \overline{B_{k-1} \cup A_k} = \overline{B}_{k-1}\overline{A}_k\ ,\ \ k = 1, 2, \ldots, n\,.$$

For $B_o = \emptyset$, we have

$$\overline{B}_1 = \overline{A}_1$$
$$\overline{B}_2 = \overline{B}_1\overline{A}_2 = \overline{A}_1\overline{A}_2$$
$$\overline{B}_3 = \overline{B}_2\overline{A}_3 = \overline{A}_1\overline{A}_2\overline{A}_3$$
$$\ldots$$

and

$$\overline{B}_n = \overline{\bigcup_{j=1}^{n} A_j} = \bigcap_{j=1}^{n} \overline{A}_j$$

(b) Use the same procedure as above by letting $C_k = \bigcap_{j=1}^{k} A_j$ and using the fourth relation in Eqs. (2.10).

2.4 (a) $S \cup C = \{1, 2, \cdots, 10\}$
(b) $A \cup B = \{1, 3, 4, 5, 6\}$
(c) $\overline{A}C = \{2, 7\}$
(d) $\overline{A} \cup (BC) = \{2, 4, 6, 7, 8, 9, 10\}$
(e) $\overline{ABC} = \{1, 2, \cdots, 10\}$
(f) $\overline{\bar{A}\bar{B}} = A \cup B = \{1, 3, 4, 5, 6\}$
(g) $(AB) \cup (BC) \cup (CA) = \{1, 5\}$

2.5 (a) $S \cup C = \{x : 0 \le x \le 10\}$
(b) $A \cup B = \{x : 1 \le x \le 6\}$
(c) $\overline{A}C = \{x : 5 < x \le 7\}$
(d) $\overline{A} \cup (BC) = \{x : 0 \le x < 1$ and $2 \le x \le 10\}$
(e) $\overline{ABC} = \{x : 0 \le x < 2$ and $5 < x \le 10\}$
(f) $\overline{\bar{A}\bar{B}} = \{x : 1 \le x \le 6\}$
(g) $(AB) \cup (BC) \cup (CA) = \{x : 1 \le x \le 6\}$

2.6

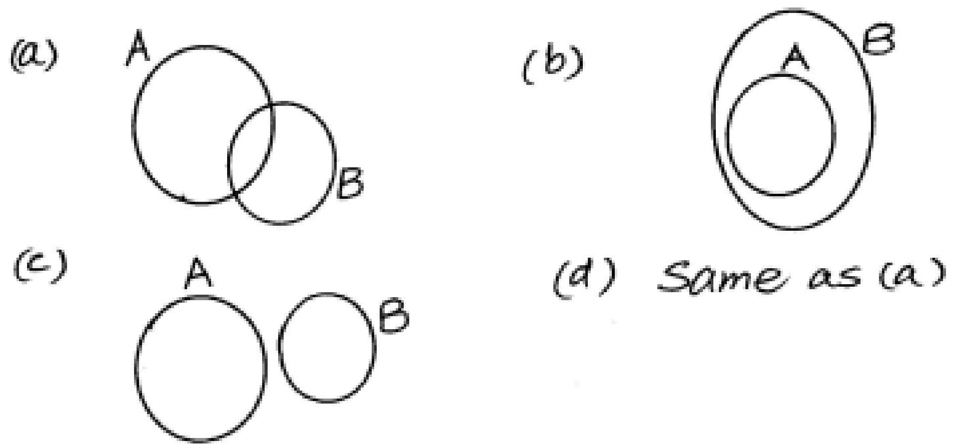


Figure 2.6

2.7 (a) $\overline{A \cup B \cup C}$ or $\bar{A}\bar{B}\bar{C}$
(b) $A(\overline{B \cup C})$ or $A\bar{B}\bar{C}$
(c) $(A\bar{B}\bar{C}) \cup (\bar{A}B\bar{C}) \cup (\bar{A}\bar{B}C)$
(d) $A \cup B \cup C$
(e) $(AB\overline{C}) \cup (A\overline{B}C)$
(f) $\overline{A}BC$
(g) $(AB) \cup (BC) \cup (CA)$
(h) $\overline{ABC}$
(i) ABC

2.8 (a) $P(AB) = P(A)P(B) = ab$

(b) $P(A \cup B) = P(A) + P(B) - P(AB) = a + b - ab$
(c) $P(A \cup B|B) = P[(A \cup B) \cap B]/P(B) = P(B)/P(B) = 1$
(d) $P(A \cup B|C) = P(A \cup B) = a + b - ab$

2.9 (a) $A \cup B$, (b) $A\overline{B} \cup \overline{A}B$

2.10 (a)
$$\begin{aligned} P(B) &= P(B|A)P(A) + P(B|\overline{A})P(\overline{A}) \\ &= 0.85(0.8) + 0.75(0.2) = 0.83 \end{aligned}$$

(b)
$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(AB) \\ &= P(A) + P(B) - P(B|A)P(A) \\ &= 0.8 + 0.83 - 0.85(0.8) = 0.95 \end{aligned}$$

(c)
$$\begin{aligned} P(A|B) &= P(B|A)P(A)/P(B) \\ &= 0.85(0.8)/0.83 = 0.82 \end{aligned}$$

(d)
$$\begin{aligned} P(A|\overline{AB}) &= P(A \cap \overline{AB})/P(\overline{AB}) \\ &= P(A\overline{B})/P(\overline{A} \cup \overline{B}) \\ &= P(\overline{B}|A)P(A)/[P(\overline{A}) + P(\overline{B}) - P(\overline{B}|\overline{A})P(\overline{A})] \\ &= 0.15(0.8)/[0.2 + 0.17 - 0.25(0.2)] \\ &= 0.375 \end{aligned}$$

(e) $P(AB) = P(B|A)P(A) = 0.85(0.8) = 0.6$
$P(A)P(B) = 0.8(0.83) = 0.664$
Since $P(AB) \neq P(A)P(B), A$ and B are not independent.

(f) $P(AB) = 0.6 \neq 0$. Hence, A and B are not mutually exclusive.

2.11 Let A: engine successful; B: computer successful; and S: satellite successful. Then,
$P(\overline{A}) = 0.008$, $P(\overline{B}) = 0.001$
$P(\overline{S}|\overline{A}) = 0.98$, $P(\overline{S}|\overline{B}) = 0.45$,
$\overline{A}$ and $\overline{B}$ are mutually exclusive and exhaustive.

(a)
$$\begin{aligned} P(\overline{S}) &= P(\overline{S}|\overline{A})P(\overline{A}) + P(\overline{S}|\overline{B})P(\overline{B}) \\ &= 0.98(0.008) + 0.45(0.001) \\ &= 0.00829 \end{aligned}$$

(b) $P(\bar{S}\bar{A}) = P(\overline{S}|\overline{A})P(\overline{A}) = 0.98(0.008) = 0.00784$

(c) $P(\overline{S}|\overline{A}) = 0.98$

2.12 Let $D = B \cup C$ and use Eq. (2.12)
$$P(A \cup B \cup C) = P(A \cup D) = P(A) + P(D) - P(AD)$$
But $P(D) = P(B \cup C) = P(B) + P(C) - P(BC)$
$$\begin{aligned} P(AD) = P(AB \cup AC) &= P(AB) + P(AC) - P(ABAC) \\ &= P(AB) + P(AC) - P(ABC) \end{aligned}$$
Hence, $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(AB) - P(BC) - P(AC) + P(ABC)$

2.13 $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 A_2) \leq P(A_1) + P(A_2)$
$$\begin{aligned} P(A_1 \cup A_2 \cup A_3) &= P(A_1 \cup A_2) + P(A_3) - P[(A_1 \cup A_2)A_3] \\ &\leq P(A_1) + P(A_2) + P(A_3) - P[(A_1 \cup A_2)A_3] \\ &\leq P(A_1) + P(A_2) + P(A_3) \end{aligned}$$
and so on.

2.14 Let A_j, $= 1, 2$, be the event that jth part is good.

(a) $P(A_1A_2) = P(A_2|A_1)P(A_1) = (14/19)(15/20) = 0.553$

(b) $P(\overline{A}_1\overline{A}_2) = P(\overline{A}_2|\overline{A}_1)P(\overline{A}_1) = (4/19)(5/20) = 0.053$

(c)
$$\begin{aligned} P(\overline{A}_1A_2 \cup A_1\overline{A}_2) &= P(\overline{A}_1A_2) + P(A_1\overline{A}_2) \\ &= P(A_2|\overline{A}_1)P(\overline{A}_1) + P(\overline{A}_2|A_1)P(A_1) \\ &= (15/19)(5/20) + (5/19)(15/20) = 0.395 \end{aligned}$$

2.15
$$\begin{aligned} \text{System Reliability } &= P(ABC) = P(A)P(B)P(C) \\ &= (0.96)(0.95)(0.95) = 0.866 \end{aligned}$$

2.16 Let $A_j, j = 1, 2, \ldots, 1000$, be the event that jth component is good, and let $P(A_j) = P(A)$. Then

$$\begin{aligned} 0.9 = P\left(\bigcap_{j=1}^{1000} A_j\right) &= P(A_1)P(A_2)\ldots P(A_{1000}) \\ &= [P(A)]^{1000} \end{aligned}$$

or

$$P(A) = 0.9^{1/1000} = 0.9999$$

2.17 Probability of failure $= P(\bar{A}\bar{B}) = P(\overline{A})P(\overline{B}) = 0.05^2 = 0.0025$
Reliability $= 1 - 0.0025 = 0.9975$

2.18 (a)
$$\begin{aligned} \text{Reliability} &= P[A(B \cup C)] = P(A)P(B \cup C) \\ &= P(A)[1 - P(\bar{B}\bar{C})] \\ &= 0.90[1 - (0.15)(0.10)] \\ &= 0.8865 \end{aligned}$$

(b) Reliability $= P[(A \cup C)(B \cup D)] = P(A \cup C)P(B \cup D)$

$P(A \cup C) = 1 - P(\bar{A})P(\bar{C}) = 1 - (1 - P_A)(1 - P_C)$

$P(B \cup D) = 1 - P(\bar{B})P(\bar{D}) = 1 - (1 - P_B)(1 - P_D)$

Reliability $= [1 - (1 - P_A)(1 - P_C)][1 - (1 - P_B)(1 - P_D)]$

2.19 Let $A_j, j = 1, 2, \ldots$, be the event that jth shot is a hit.

(a) $P(\overline{A}_1\overline{A}_2) = P(\overline{A}_1)P(\overline{A}_2) = 0.1^2 = 0.01$

(b)
$$\begin{aligned} &P(\overline{A}_1\overline{A}_2\overline{A}_3A_4 \cup \overline{A}_1\overline{A}_2\overline{A}_3\overline{A}_4A_5 \cup \overline{A}_1\overline{A}_2\overline{A}_3\overline{A}_4\overline{A}_5A_6) \\ &= P(\overline{A}_1\overline{A}_2\overline{A}_3A_4) + P(\overline{A}_1\overline{A}_2\overline{A}_3\overline{A}_4A_5) + P(\overline{A}_1\overline{A}_2\overline{A}_3\overline{A}_4\overline{A}_5A_6) \\ &= (0.1)^3(0.9) + (0.1)^4(0.9) + (0.1)^5(0.9) \\ &= 0.000999 \end{aligned}$$

2.20 No

2.21 (a) $P(A \cup B) = P(A) + P(B) - P(A)P(B)$

$0.7 = 0.4 + P(B) - 0.4P(B)$

$P(B) = 0.5$

(b) $P(A \cup B) = P(A) + P(B)$

$0.7 = 0.4 + P(B)$

$P(B) = 0.3$

2.22 No.

(a) $P(A \cup B) = P(A) + P(B) - P(AB)$ gives $0.75 = P(A) + P(B) - 0.25$
$P(AB) = P(A)P(B)$ gives $0.25 = P(A)P(B)$

Solving the above equations simultaneously, we have

$$P(A) = P(B) = 0.5$$

(b) Mutual exclusiveness implies $P(AB) = 0$, but $P(AB) = 0.25$. Hence, it is impossible.

2.23 Under condition of mutual exclusiveness,
(a) False (b) True (c) False (d) True (e) False
Under condition of independence,
(a) True (b) False (c) False (d) False (e) True

2.24 Let A be the event of accident due to human error in any given minute, and B be the event of accident due to mechanical breakdown in any given minute.

(a)
$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(AB) \\ &= 10^{-5} + 10^{-7} - (10^{-5})(10^{-7}) \\ &\cong 10^{-5} \end{aligned}$$

(b) Yes if $P(A)P(B) \ll P(A)$ or $P(B)$, which is the case here.

(c)
$$\begin{aligned} P(\bar{A}\bar{B}\bar{A}\bar{B}\ldots\bar{A}\bar{B}) &= [P(\overline{A})]^{60\times24\times365}[P(\overline{B})]^{60\times24\times365} \\ &= [(1-10^{-5})(1-10^{-7})]^{(60\times24\times365)} \\ &\cong 0.00499 \end{aligned}$$

2.25

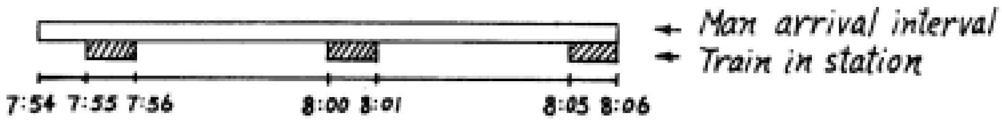


Figure 2.25

Prob. $= 3/12 = 0.25$.

2.26 Same situation as Prob. 2.25.

(a) $P(t_0 \le T \le t_1) = \frac{t_1 - t_0}{t}$

(b)
$$\begin{aligned} P(t_0 \le T \le t_1 | T \ge t_0) &= \frac{P(t_0 \le T \le t_1 \cap T \ge t_0)}{P(T \ge t_0)} \\ &= \frac{P(t_0 \le T \le t_1)}{P(T \ge t_0)} = \frac{t_1 - t_0}{t - t_0} \end{aligned}$$

2.27 (a) $P(AB) = P(A) + P(B) - P(A \cup B) = 0.6 + 0.6 - 0.7 = 0.5$
(b) $P(C) = 0.7$
(c) Prob. $= P(C) - P(A) = 0.1$
(d) Prob. $= \frac{P(AB)}{P(A)} = \frac{0.5}{0.6} = 0.833$
(e) Prob. $= \frac{P(A)}{P(C)} = \frac{0.6}{0.7} = 0.857$

2.28 Let A_j be the event that the jth gap is acceptable. Then, $P(A_j) = 0.65$ for all j. It is also safe to assume that all A_j's are independent.

(a) $P(\overline{A}_1) = 0.35$

(b) $P(\overline{A}_1\overline{A}_2) = P(\overline{A}_1)(\overline{A}_2) = (0.35)(0.35) = 0.1225$
(c) $P(A_2|A_1) = P(A_1A_2)/P(A_1) = P(A_1)P(A_2)/P(A_1) = P(A_2) = 0.65$

2.29 Let A be the event of a part functioning properly and $B_j, j = 1,2,3$, be the event that the part comes from plant j.

$$\begin{aligned} P(A) &= P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + P(A|B_3)P(B_3) \\ &= (0.2)(0.25) + (0.3)(0.5) + (0.4)(0.25) \\ &= 0.3 \end{aligned}$$

2.30 Let A be the event of system failure and $B_j, j = 1,2,3$, be the events, respectively, of low, medium and high demand levels.

(a) $$\begin{aligned} P(A) &= P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + P(A|B_3)P(B_3) \\ &= (0.0)(0.6) + (0.1)(0.3) + (0.5)(0.1) \\ &= 0.08 \end{aligned}$$

(b) $P(B_2|A) = \frac{P(A|B_2)P(B_2)}{P(A)} = \frac{(0.1)(0.3)}{0.08} = 0.375$

2.31 Let A be the event that test is positive (having cancer) and B be the event that individual has cancer. Then

$$P(B) = 0.005$$

$$P(A|B) = 0.95\ ,\ P(A|\overline{B}) = 0.05$$

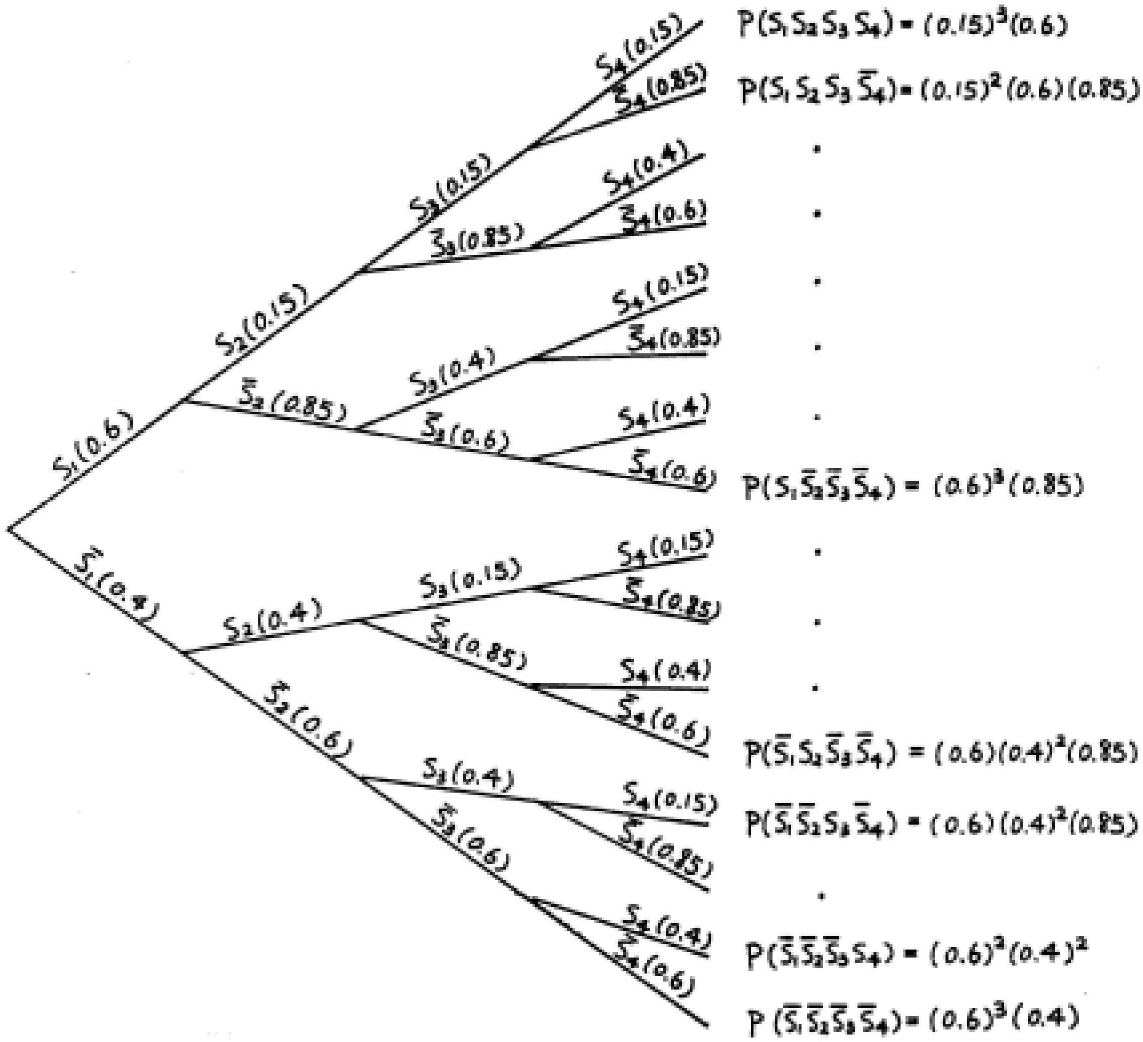
$$\begin{aligned} P(B|A) &= \frac{P(A|B)P(B)}{P(A)} = \frac{P(A|B)P(B)}{P(A|B)P(B) + P(A|\overline{B})P(\overline{B})} \\ &= \frac{(0.95)(0.005)}{(0.95)(0.005) + (0.05)(0.995)} = 0.087 \end{aligned}$$

2.32 Let $S_j, j = 1,2,3$, be the events, respectively, of unacceptable, marginal and acceptable qualities.

(a) $P(AS_1) = P(S_1|A)P(A) = (128/140)(140/365) = 0.351$
(b) $P(S_1|C) = 110/120 = 0.917$
(c) $P(B|S_2) = \frac{P(S_2|B)P(B)}{P(S_2)} = \frac{(5/105)(105/365)}{20/365} = 0.25$

2.33 $P(A_2|A_1)P(A_1) = P(A_1A_2)$
$P(A_3|A_1A_2)P(A_1A_2) = P(A_1A_2A_3)$

2.34 (a) $P(S_1S_2S_3S_4) = (0.15)^3(0.6) = 0.002$
(b) $P(\overline{S}_1\overline{S}_2\overline{S}_3\overline{S}_4) = (0.6)^3(0.4) = 0.086$
(c) $$\begin{aligned} & P(\overline{S}_1\overline{S}_2\overline{S}_3\overline{S}_4) + P(\overline{S}_1\overline{S}_2\overline{S}_3S_4) + P(\overline{S}_1\overline{S}_2S_3\overline{S}_4) + P(\overline{S}_1S_2\overline{S}_3\overline{S}_4) + P(S_1\overline{S}_2\overline{S}_3\overline{S}_4) \\ &= 0.086 + (0.6)^2(0.4)^2 + (0.6)(0.4)^2(0.85) + (0.6)(0.4)^2(0.85) + (0.6)^3(0.85) \\ &= 0.4904 \end{aligned}$$


$S_1(0.6)$
$\bar{S}_1(0.4)$
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$\bar{S}_2(0.85)$
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$\bar{S}_4(0.6)$
$P(S_1S_2S_3S_4) = (0.15)^3(0.6)$
$P(S_1S_2S_3\bar{S}_4) = (0.15)^2(0.6)(0.85)$
$P(S_1\bar{S}_2\bar{S}_3\bar{S}_4) = (0.6)^3(0.85)$
$P(\bar{S}_1S_2\bar{S}_3\bar{S}_4) = (0.6)(0.4)^2(0.85)$
$P(\bar{S}_1\bar{S}_2S_3\bar{S}_4) = (0.6)(0.4)^2(0.85)$
$P(\bar{S}_1\bar{S}_2\bar{S}_3S_4) = (0.6)^2(0.4)^2$
$P(\bar{S}_1\bar{S}_2\bar{S}_3\bar{S}_4) = (0.6)^3(0.4)$

Figure 2.34